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Exterior problem for the Boltzmann equation with temperature difference: asymptotic stability of steady solutions

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Abstract

The asymptotic stability of non-equilibrium steady solutions to the exterior problem for the Boltzmann equation was first proved by Ukai and Asano in [17], under the assumption that the temperature associating with the far-field Maxwellian and the one preserved by the kinetic boundary condition are the same. In this paper, we generalize Ukai and Asano's result in the sense that the two temperatures mentioned above are allowed to be different. The proof of the main theorem is based on the ideas developed in [16,17] and [20] as well as some new observations.

Keywords: Exterior problem; Boltzmann equation; Temperature difference; Steady solutions; Asymptotic stability

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1. Introduction

The flow past an obstacle is one of the most classical and important subjects in gas dynamics and fluid mechanics. However, the mathematical problems in this direction are extremely challenging and rigorous theories are absent for most of the general cases. At the macroscopic level, attempts have been made by many outstanding mathematicians with a lot of impressive results (e.g. Bers [2,3], Courant–Friedrichs [5], Dong [6], Finn–Gilbarg [8,9], Gilbarg–Shiffman [10], Morawetz [12] and the references therein), while at the mesoscopic level, that is, for kinetic equations (e.g. Boltzmann equation), the mathematical understanding is very limited.

In their pioneering work [16], Ukai and Asano gave the first rigorous but technical proof on the existence of a steady gas flow past a bounded convex obstacle in the context of the Boltzmann equation. More precisely, they showed that if the gas is in equilibrium at the space infinity, moving with a prescribed constant (but sufficiently small) bulk velocity, then the corresponding exterior problem will admit a unique non-equilibrium steady solution, which is close to the given profile at the space infinity (i.e., the far-field Maxwellian). Later, in a continuation paper (see [17]), they further proved that such a non-equilibrium steady solution is asymptotically stable in time.

However, the assumptions in both [16] and [17] seem too strong to include some physically interesting situations. In particular, it is required in [16] and [17] that the temperature preserved by the kinetic boundary condition (denoted by T_w) and the one associating with the far-field Maxwellian (denoted by T_∞) are the same. Several years ago, Ukai, Yang and Zhao considered (see [20]) a more general setting, in which the temperatures T_w and T_∞ are allowed to be different. Roughly speaking, they proved that as long as the bulk velocity at the space infinity and the temperature difference $T_w - T_\infty$ are sufficiently small, the exterior problem of the Boltzmann equation will admit a unique non-equilibrium steady solution. This result generalizes the previous work in [16].

Then, a natural question is to ask whether the steady solution constructed in [20] is asymptotically stable or not. In this paper, we will give an affirmative answer to this problem.

1.1. The mathematical formulation

Throughout this paper, we always assume that the space dimension $n \geq 3$. Let $F := F(t, x, \xi)$ be the distribution function of the gas particles at time $t \geq 0$, around position $x \in \Omega$, with velocity $\xi \in \mathbb{R}^n$. Then, the gas flow past an obstacle in the context of the Boltzmann equation can be formulated by the following exterior problem:

$$\begin{cases} \partial_t F + \xi \cdot \nabla_x F = \mathcal{Q}(F, F) \quad \text{in } (0, \infty) \times Q, \\ \gamma^- F = \mathbb{B}\gamma^+ F \quad \text{in} \quad (0, \infty) \times S^-, \\ \lim_{|x| \to \infty} F = \mu_{c,T} \quad \text{in} \quad (0, \infty) \times \mathbb{R}^n_\xi, \\ F(0, x, \xi) = F_0(x, \xi) \quad \text{in} \quad Q. \end{cases} \tag{1}$$