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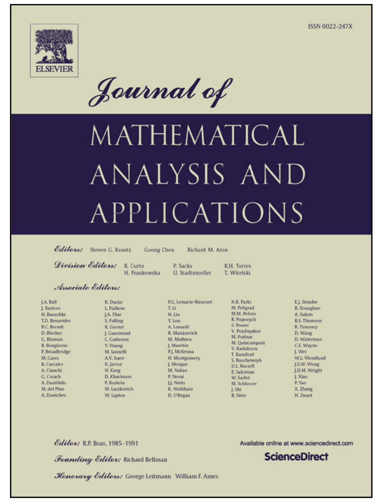

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Strong Metric Subregularity of Mappings in Variational Analysis and Optimization

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Dedicated to the memory of Jonathan M. Borwein

Abstract

Although the property of strong metric subregularity of set-valued mappings has been present in the literature under various names and with various (equivalent) definitions for more than two decades, it has attracted much less attention than its older "siblings", the metric regularity and the strong (metric) regularity. The purpose of this paper is to show that the strong metric subregularity shares the main features of these two most popular regularity properties and is not less instrumental in applications. We show that the strong metric subregularity of a mapping F acting between metric spaces is stable under perturbations of the form $f + F$, where f is a function with a small calmness constant. This result is parallel to the Lyusternik-Graves theorem for metric regularity and to the Robinson theorem for strong regularity, where the perturbations are represented by a function f with a small Lipschitz constant. Then we study perturbation stability of the same kind for mappings acting between Banach spaces, where f is not necessarily differentiable but admits a set-valued derivative-like approximation. Strong metric q-subregularity is also considered, where q is a positive real constant appearing as exponent in the definition. Rockafellar's criterion for strong metric subregularity involving injectivity of the graphical derivative is extended to mappings acting in infinite-dimensional spaces. A sufficient condition for strong metric subregularity is established in terms of surjectivity of the Fréchet coderivative, and it is shown by a counterexample that surjectivity of the limiting coderivative is not a sufficient condition for this property, in general. Then various versions of Newton's method for solving generalized equations are considered including inexact and semismooth methods, for which superlinear convergence is shown under strong metric subregularity. As applications to optimization, a characterization of the strong metric subregularity of the KKT mapping is obtained, as well as a radius theorem for the optimality mapping of a nonlinear programming problem. Finally, an error estimate is derived for a discrete approximation in optimal control under strong metric subregularity of the mapping involved in the Pontryagin principle.

Key Words. strong metric subregularity, perturbations and approximations, generalized derivatives, Newton's method, nonlinear programming, optimal control.

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