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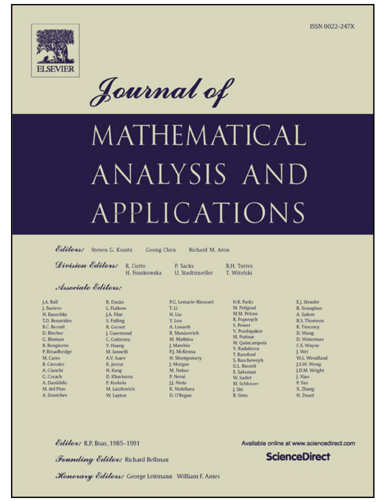

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Smoothness of the Metric Projection onto Nonconvex Bodies in Hilbert Spaces*

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Abstract

Based on a fundamental work of R. B. Holmes from 1973, we study differentiability properties of the metric projection onto prox-regular sets. We show that if the set is a nonconvex body with a C^{p+1}-smooth boundary, then the projection is $\mathcal{C}^p$-smooth near suitable open truncated normal rays, which are determined only by the function of prox-regularity. A local version of the same result is established as well, namely, when the smoothness of the boundary and the prox-regularity of the set are assumed only near a fixed point. Finally, similar results are derived when the prox-regular set is itself a C^{p+1}-submanifold.

Key words: distance function, metric projection, nonconvex body, prox-regular set, normal cone, submanifold.

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1 Introduction

In his 1973 fundamental paper [13], R. B. Holmes showed that, whenever we have a closed convex set K in a Hilbert space X such that

(i) K has nonempty relative interior (namely, the interior of K as a subset of $Y = \overline{\mathrm{aff}}(K)$ is nonempty), and

(ii) the boundary of K as a subset of Y, $\mathrm{bd}\, K$, is a $\mathcal{C}^{p+1}$-submanifold at a point $x_0 \in \mathrm{bd}\, K$, where p is a positive integer,

then the metric projection P_K is a mapping of class $\mathcal{C}^p$ in an open neighborhood W of the open normal ray

$$\mathrm{Ray}_{x_0}(K) := \{x_0 + t\nu \ : \ t > 0\},$$

where ν denotes the unit exterior normal vector of K at x_0. The main steps of his approach to arrive to this theorem were:

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