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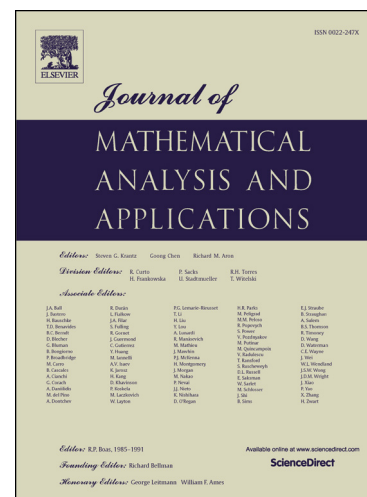
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# Smoothness of the Metric Projection onto Nonconvex Bodies in Hilbert Spaces\*

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## Abstract

Based on a fundamental work of R. B. Holmes from 1973, we study differentiability properties of the metric projection onto prox-regular sets. We show that if the set is a nonconvex body with a  $C^{p+1}$ -smooth boundary, then the projection is  $C^p$ -smooth near suitable open truncated normal rays, which are determined only by the function of prox-regularity. A local version of the same result is established as well, namely, when the smoothness of the boundary and the prox-regularity of the set are assumed only near a fixed point. Finally, similar results are derived when the prox-regular set is itself a  $C^{p+1}$ -submanifold.

**Key words:** distance function, metric projection, nonconvex body, prox-regular set, normal cone, submanifold.

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## 1 Introduction

In his 1973 fundamental paper [13], R. B. Holmes showed that, whenever we have a closed convex set  $K$  in a Hilbert space  $X$  such that

- (i)  $K$  has nonempty relative interior (namely, the interior of  $K$  as a subset of  $Y = \overline{\text{aff}}(K)$  is nonempty), and
- (ii) the boundary of  $K$  as a subset of  $Y$ ,  $\text{bd } K$ , is a  $C^{p+1}$ -submanifold at a point  $x_0 \in \text{bd } K$ , where  $p$  is a positive integer,

then the metric projection  $P_K$  is a mapping of class  $C^p$  in an open neighborhood  $W$  of the open normal ray

$$\text{Ray}_{x_0}(K) := \{x_0 + t\nu : t > 0\},$$

where  $\nu$  denotes the unit exterior normal vector of  $K$  at  $x_0$ . The main steps of his approach to arrive to this theorem were:

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