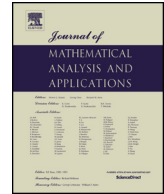




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Self-contracted curves in Riemannian manifolds

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ABSTRACT

It is established that every self-contracted curve in a Riemannian manifold has finite length, provided its image is contained in a compact set.

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1. Introduction

This work is devoted to the study of self-contracted curves on Riemannian manifolds \mathcal{M} .

Definition 1.1 (*Self-contracted curve*). Let \mathcal{M} be a Riemannian manifold and let d_g denote its geodesic distance. Given an interval $I = [0, T_\infty)$ with $T_\infty \in (0, \infty) \cup \{\infty\}$, a curve $\gamma : I \rightarrow \mathcal{M}$ is called *self-contracted*, if for every $t_1 \leq t_2 \leq t_3$ in I we have

$$d_g(\gamma(t_1), \gamma(t_3)) \geq d_g(\gamma(t_2), \gamma(t_3)). \tag{1.1}$$

In other words, for every $\tau \in [0, T_\infty)$ the function $t \mapsto d_g(\gamma(t), \gamma(\tau))$ is nonincreasing on $[0, \tau]$.

Self-contracted curves were introduced in [3, Definition 1.2.]. The motivation of this definition comes from the following example.

Example 1.2. If $f : \mathbb{R}^n \rightarrow \mathbb{R}^+$ is a \mathcal{C}^1 -smooth convex function and if $\gamma : (0, +\infty) \rightarrow \mathbb{R}^n$ is smooth and satisfies $\gamma'(t) = -\nabla f(\gamma(t))$ for all $t > 0$, then γ is a self-contracted curve.

Indeed, observe first that $(f(\gamma(t)))' = -\|\nabla f(\gamma(t))\|^2 \leq 0$, thus the function $t \mapsto f(\gamma(t))$ is nonincreasing. Therefore, since f is convex, if $\tau \geq t$, then

$$\frac{d}{dt} \left(\frac{1}{2} \|\gamma(\tau) - \gamma(t)\|^2 \right) = \langle \gamma(\tau) - \gamma(t), \nabla f(\gamma(t)) \rangle \leq f(\gamma(\tau)) - f(\gamma(t)) \leq 0.$$

This proves that the function $t \mapsto \|\gamma(t) - \gamma(\tau)\|$ is nonincreasing on $[0, \tau]$.

One of the main interests in studying self-contracted curves lies in its applications. Rectifiability of self-contracted curves has been applied in different areas, including continuous and discrete dynamical systems, optimization and convergence of algorithms. See for example [3] and [4].

The definition of self-contractedness is purely metric: if φ is a nondecreasing function from an interval J onto I , then $\gamma \circ \varphi$ is also self-contracted, so this notion does not depend on the particular parametrization of the oriented graph $\{\gamma(t); t \in I\}$. Self-contractedness does not require prior smoothness or continuity assumption on the curve as shown by the following example.

Example 1.3. Let $\gamma : \mathbb{R} \rightarrow \mathbb{C}$ defined by $\gamma(t) = t$ if $t \leq -1$, $\gamma(t) = -t$ if $-1 < t \leq 0$ and $\gamma(t) = it$ if $t > 0$. The curve γ is self-contracted, is not smooth at $t = 0$, is discontinuous at $t = -1$, and moreover does not admit a continuous self-contracted extension, *i.e.* there exists no continuous self-contracted curve $\Gamma : \mathbb{R} \rightarrow \mathbb{C}$ such that $\{\Gamma(t) : t \in \mathbb{R}\} \supset \{\gamma(t) : t \in \mathbb{R}\}$.

In a Euclidean setting it has been established in [4, Section 3] (and independently in [8] for continuous curves) that bounded self-contracted curves have finite length. In both cases the proof was based on an old

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