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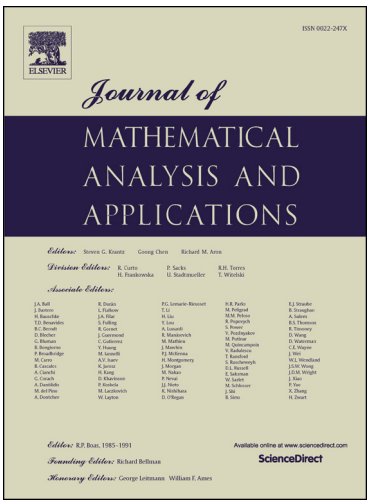


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Valeriya Lykina, Sabine Pickenhain

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# Pontryagin Type Maximum Principle for Budget-Constrained Infinite Horizon Optimal Control Problems with Linear Dynamics[1]

Valeriya Lykina[a], Sabine Pickenhain[b]

[a] *Vienna University of Technology, Austria; e-mail address: valeriya.lykina@tuwien.ac.at*
[b] *Brandenburg University of Technology Cottbus, Germany; e-mail address: Sabine.Pickenhain@b-tu.de*

**Abstract**

In this paper a class of infinite horizon optimal control problems with an isoperimetrical constraint, also interpreted as a budget constraint, is considered. Herein a linear both in the state and in the control dynamic is allowed. The problem setting includes a weighted Sobolev space as the state space. For this class of problems, we establish the necessary optimality conditions in form of a Pontryagin Type Maximum Principle including a transversality condition. The proved theoretical result is applied to a linear-quadratic regulator problem.

*Keywords:* budget constraint, infinite horizon, optimal control, Pontryagin Type Maximum Principle, weighted Sobolev spaces, linear-quadratic regulator.

## 1. Introduction

Infinite horizon optimal control problems have been investigated since the seventies of the last century, cf. [2], [3], [4], [7], [13], [14] among many others. This class of problems finds many applications in the economics, biology and stabilization problems as well, cf. [11], [10]. It turned out that in many models it is not possible to assure the existence of an optimal solution, cf. [20] and [23]. One way to overcome this difficulty is to introduce an additional pointwise state constraint which has an essential impact on the boundedness of the feasible set and on the semicontinuity of the functional in the objective and therefore guarantees the existence of optimal solution, cf. [20] and [23]. However, this constraint may prescribe in a strong manner how the optimal solution has to look like which seems to be rather restrictive. In this paper we omit the pointwise state constraint and pose a weaker isoperimetric constraint instead, also referred to as a budget constraint. Throughout the paper, the latter is assumed to be linear in the state variable. The present class of problems was already introduced in [21], where some existence results were obtained. The main task of the present paper is to establish a Pontryagin Type Maximum Principle as a necessary optimality condition which contains a transversality condition. The transversality conditions are a well known bottleneck of infinite horizon problems. An example of Halkin, see [14], shows that the classical transversality condition $y(T) = 0$, which has to be necessarily satisfied by the adjoint function $y(\cdot)$ in case of a control problem with finite fixed horizon, cannot be automatically extended to the case of an optimal control problem with infinite horizon. It means that the limiting condition

$$\lim_{t\to\infty} y(t) = 0 \tag{1}$$

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