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Second order sweeping process with a Lipschitz perturbation

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ABSTRACT

In this paper we prove a theorem concerning the existence of solutions for a second order sweeping process with a Lipschitz single valued perturbation in Hilbert space.

1. Introduction

Let H be a real Hilbert space, $T > 0$ and let $F : [0, T] \times H \rightrightarrows H$ be a set-valued map with nonempty convex compact values satisfying the linear compact growth condition

$$F(t, x) \subset \beta(t)(1 + \|x\|)L, \ \forall (t, x) \in [0, T] \times H,$$

where $\beta(.) \in \mathbf{L}^1([0, T], \mathbb{R})$ is a non-negative function and L is a fixed strongly compact subset of H. Assume that F is upper semicontinuous with respect to the second variable and for each $x \in H$, $F(., x)$ has a Lebesgue-measurable selection. Under these assumptions, the authors proved in [14] that the perturbed sweeping process

$$(\mathcal{Q}) \ \begin{cases} -\dot{u}(t) \in N_{K(t)}(u(t)) + F(t, u(t)), \ a.e. \ t \in [0, T], \\ u(0) = u_0 \in K(0); \end{cases}$$

has at least one absolutely continuous solution whenever H is separable and the sets $K(t)$ are r-prox-regular ($r > 0$).

Problem $(\mathcal{Q})$ has been introduced and perfectly studied by Moreau in a series of papers (see [17–21]) when $F \equiv \{0\}$ and the sets $K(t)$ are convex, and he had showed that this differential inclusion is a model

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for several practical situations in mechanics, such as water falling in a cavity, the dynamics of systems with perfect unilateral constraints, plasticity and the evolution of elastoplastic systems. We refer to the book of Monteiro Marques [16] for a well unified presentation. Also in this case, the authors in [12] and [26] gave in the infinite dimensional Hilbert space some results when for all t, $K(t)$ belongs to the more general class of prox-regular sets. For perturbations $F \not\equiv \{0\}$, we can refer the reader to [7] for the continuous case, and the first work in the discontinuous case is due to Castaing and Monteiro Marques [9]. After, the same result has been established by Thibault [26] and the references in [26].

In a different context, Kunze and Monteiro Marques studied the following state dependent sweeping process

$$\begin{cases} -\dot{u}(t) \in N_{K(t,u(t))}(u(t)), \ a.e. \ t \in [0,T], \\ u(0) = u_0 \in K(0,u_0). \end{cases}$$

This type of problems is used in micromechanical damage models (the so-called Gurson-models) for iron materials with memory to describe the evolution of the quasi strain in presence of small damages, see [15].

In [13], the authors proved the existence and uniqueness of a solution for $(\mathcal{Q})$ whenever F is a single-valued map, say f, which is Lipschitz continuous with respect to the second variable on any bounded subset of H and which satisfies the natural growth condition

$$\|f(t,x)\| \leq \beta(t)(1 + \|x\|), \ \forall (t,x) \in [0,T] \times H.$$

See also [3], in which the authors have combined the two differential inclusions studied in [13] and [14].

In our present paper, we generalize the same result to the second order case, that is, we prove that the problem

$$(\mathcal{P}_f) \ \begin{cases} -\ddot{x}(t) \in N_{K(x(t))}(\dot{x}(t)) + f(t, x(t), \dot{x}(t)), \ a.e. \ t \in [0,T]; \\ \dot{x}(t) \in K(x(t)), \ \forall t \in [0,T]; \\ x(0) = x_0; \ \dot{x}(0) = u_0, \end{cases}$$

has a Lipschitz solution under the assumptions: the set-valued map $K(.)$ takes r-prox-regular values, moves in a Lipschitz way, and f is Lipschitz with respect to the second and third variables and satisfies the natural growth condition for some real number $c > 0$, that is,

$$\|f(t,x,u)\| \leq c(1 + \|x\| + \|u\|), \ \forall (t,x,u) \in I \times H \times H.$$

To the best of our knowledge, the first work in the second order sweeping process $(\mathcal{P}_f)$ when $f = 0$, is due to Castaing [6], and in [25] the author studied (see [16]) a problem which contains the particular case of the inclusion

$$-\ddot{u}(t) \in N_K(u(t))$$

where K is a fixed convex set. This inclusion is the problem of frictionless elastic shocks in the region K.

Our motivation here is to consider the case where such problems are submitted to various perturbations which can be represented in the form of the differential inclusion $(\mathcal{P}_f)$, and we refer the reader to the book of Monteiro Marques [16] for a detailed study of $(\mathcal{P}_f)$ where the perturbation f not depends on the velocity and the set $K(x)$ is a tangent cone.

For the second order sweeping process with set-valued perturbations we refer the reader to [1,2,4,7,8].

The paper is organized as follows. Section 2 is devoted to some definitions and notation needed later. In section 3, we prove our main result.