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Vanishing viscosity limit of the radiation hydrodynamic equations with far field vacuum

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We prove the vanishing viscosity limit of the Navier–Stokes–Boltzmann equations (see (1.3)) to the Euler–Boltzmann equations (see (1.9)) for a three-dimensional compressible isentropic flow in radiation hydrodynamics. It is shown that under some reasonable assumptions for the radiation coefficients, there exists a unique regular solution of Navier–Stokes–Boltzmann equations with degenerate viscosities, arbitrarily large initial data and far field vacuum, whose life span is uniformly positive in the vanishing viscosity limit. It is worth paying special attention to the fact that, via introducing two different symmetric structures and applying some techniques dealing with the complexity caused by the strong coupling between fluid and radiation field, we can also give some uniform estimates of $(I, \rho^{\frac{\gamma-1}{2}}, u)$ in H^3 and of $\nabla\rho/\rho$ in D^1, which provide the convergence of the regular solution of the viscous radiation flow to that of the inviscid radiation flow (see Li–Zhu [17]) in $L^\infty([0,T]; H^{s'})$ space for any $s' \in [2,3)$ with a rate of $\epsilon^{2(1-\frac{s'}{3})}$.

1. Introduction

This paper is concerned with the vanishing viscosity limit of smooth solutions to the compressible isentropic radiation hydrodynamic equations with far field vacuum in $\mathbb{R}^3$.

Usually speaking, radiation transfer is the most effective physical mechanism which affects the energy exchange in fluids, so it is worth taking effects of the radiation field into consideration in the hydrodynamic equations, which result from the balances of particles, momentum and energy. In order to describe the radiation field and its interaction with matter, at any time t, we need 6 variables to specify the state

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of a photon in phase space, namely, 3 position variables and 3 velocity (or momentum) variables. Now we denote by x the 3 position variables, and introduce the frequency v and the travel direction Ω of the photon to replace the equivalent 3 momentum variables. Then we can define distribution function as $f \equiv f(t,x,v,\Omega)$:

$$\mathrm{d}n = f(t,x,v,\Omega)\mathrm{d}x\mathrm{d}v\mathrm{d}\Omega,$$

where n is the number of photons; dn is the number of photons (at time t) at space point x in a volume element $\mathrm{d}x$, with local frequency v in a frequency interval $\mathrm{d}v$, and traveling in a direction Ω in the cubic angel element $\mathrm{d}\Omega$. In the radiation transport, we usually use the specific radiation intensity $I = I(t,x,v,\Omega)$ to replace the distribution function f. The specific radiation intensity is defined as $I = chvf(x,t,v,\Omega)$ with the Planck constant h and the light speed $c = 3 \times 10^8$ m/s.

Regarding the three basic interactions between photons and matter, namely, absorption, scattering and emission, we have transport equation in the general form

$$\frac{1}{c}\partial_t I + \Omega \cdot \nabla I = A_r, \tag{1.1}$$

where A_r is the interaction operator given by

$$A_r = S - \sigma_a I + \int_0^\infty \int_{S^2} \left(\frac{v}{v'}\sigma_s(v' \to v, \Omega' \cdot \Omega)I' - \sigma_s(v \to v', \Omega \cdot \Omega')I \right) \mathrm{d}\Omega' \mathrm{d}v', \tag{1.2}$$

S^2 is the unit sphere in $\mathbb{R}^3$, $I = I(t,x,v,\Omega)$, $I' = I(t,x,v',\Omega')$, $S = S(t,x,v,\rho,\theta)$ is the rate of energy emission due to spontaneous process, and $\sigma_a = \sigma_a(t,x,v,\rho,\theta)$ denotes the absorption coefficient that may also depend on the mass density ρ and the temperature θ of the matter. Similarly to absorption, a photon can undergo scattering interactions with matter, and the scattering interaction serves to change the photon's characteristics v' and Ω' to a new set of characteristics v and Ω. To quantitatively describe the scattering event, one requires a probabilistic statement concerning this change, which leads to the definition of the 'differential scattering coefficient' $\sigma_s(v' \to v, \Omega' \cdot \Omega) \equiv \sigma_s(v' \to v, \Omega' \cdot \Omega, \rho, \theta)$ that may depend on ρ and θ (in general, σ_s is independent of θ) such that the probability of a photon being scattered from v' to v contained in $\mathrm{d}v$, from Ω' to Ω contained in $\mathrm{d}\Omega$, and traveling a distance ds is given by $\sigma_s(v' \to v, \Omega' \cdot \Omega)\mathrm{d}v\mathrm{d}\Omega ds$. Here, if we ignore the effects caused by emission and absorption, i.e., $S = \sigma_a = 0$, then (1.3) is just the typically Boltzmann equations coming from the kinetic theory of gases [12].

Now we take radiation effect into consideration for viscous fluids to have the following isentropic Navier–Stokes–Boltzmann equations in $\mathbb{R}^3$:

$$\begin{cases} \frac{1}{c}I_t + \Omega \cdot \nabla I = A_r, \\ \rho_t + \mathrm{div}(\rho u) = 0, \\ \left(\rho u + \frac{1}{c^2}F_r\right)_t + \mathrm{div}(\rho u \otimes u + P_r) + \nabla P_m = \mathrm{div}\mathbb{T}, \end{cases} \tag{1.3}$$

where the dependence on θ is reduced into the dependence on density by the laws of Boyle and Gay-Lussac for ideal gas. We consider (1.3)'s Cauchy problem with initial data

$$I|_{t=0} = I_0(v,\Omega,x), \quad (\rho,u)|_{t=0} = (\rho_0(x), u_0(x)), \ (v,\Omega,x) \in \mathbb{R}^+ \times S^2 \times \mathbb{R}^3, \tag{1.4}$$