J. Math. Anal. Appl. 452 (2017) 990–1004

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

# Time decay rates for the compressible viscoelastic flows

Guochun Wu[a], Zhensheng Gao[a], Zhong Tan[b,*]

[a] School of Mathematical Sciences, Huaqiao University, Quanzhou 362021, China
[b] School of Mathematical Sciences, Xiamen University, Xiamen 361005, China

ARTICLE INFO

*Article history:*
Received 6 December 2016
Available online 22 March 2017
Submitted by X. Zhang

*Keywords:*
Viscoelastic flows
Global existence
Optimal decay rates

ABSTRACT

In this paper, we investigate the time decay rates of the global smooth solutions to the compressible viscoelastic flows in $\mathbb{R}^3$. The global existence and uniqueness of smooth solutions are obtained under the assumption that the $H^3$ norm of the initial data is small, but the higher order derivatives can be arbitrarily large. Moreover, if the initial data is bounded in $\dot{H}^{-s}(0 \le s < \frac{3}{2})$ or $\dot{B}^{-s}_{2,\infty}(0 < s \le \frac{3}{2})$, we establish the optimal decay rates of the smooth solutions by a regularity interpolation trick and delicate energy methods.



## 1. Introduction

In this paper, we consider the following well-known compressible viscoelastic flow for the motion of compressible non-Newtonian fluid of Oldroyd type:

$$\begin{cases} \rho_t + \operatorname{div}(\rho \mathbf{u}) = 0, \\ (\rho \mathbf{u})_t + \operatorname{div}(\rho \mathbf{u} \otimes \mathbf{u}) + \nabla P(\rho) = \operatorname{div}\mathbb{T} + a\operatorname{div}(\rho F F^T), \\ F_t + \mathbf{u} \cdot \nabla F = \nabla \mathbf{u} F, \end{cases} \tag{1.1}$$

for $(x,t) \in \mathbb{R}^3 \times \mathbb{R}^+$. Here $\rho, \mathbf{u}$ and $F \in M^{3\times 3}$ (the set of $3 \times 3$ matrices with positive determinants) denote the density, velocity and the deformation gradient respectively. $P(\rho)$ is the pressure, we will consider only polytropic fluids, so that the equation of state for the fluid is given by $P(\rho) = A\rho^\gamma$, where $A > 0$ is a constant, $\gamma > 1$ is the adiabatic exponent. $F^T$ means the transpose matrix of $F$. The stress tensor $\mathbb{T}$ is given by

$$\mathbb{T} = \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \lambda(\operatorname{div}\mathbf{u})\mathrm{I},$$

* Corresponding author.
*E-mail addresses:* guochunwu@126.com (G. Wu), gaozhensheng@hqu.edu.cn (Z. Gao), ztan85@163.com (Z. Tan).

http://dx.doi.org/10.1016/j.jmaa.2017.03.044

where I is the identity matrix. $\mu$ and $\lambda$ are the coefficient of viscosity and second coefficient of viscosity, which are assumed to satisfy the following physical condition:

$$\mu > 0, \quad 3\lambda + 2\mu > 0.$$

The positive parameter $a$ represents the speed of propagation of shear waves. The notation $\mathbf{u} \cdot \nabla F$ is understood to be $(\mathbf{u} \cdot \nabla)F$. The corresponding elastic energy for system (1.1) is the special form of the Hookean linear elasticity:

$$W(F) = \frac{a}{2}|F|^2 + \frac{1}{\rho}\int_0^\rho P(s)ds.$$

In the following discussion, initial data satisfies

$$(\rho, \mathbf{u}, F)|_{t=0} = (\rho_0(x), \mathbf{u}_0(x), F_0(x)) \to (\bar{\rho}, 0, \mathrm{I}), \quad |x| \to \infty, \tag{1.2}$$

and

$$\mathrm{div}(\rho F^T) = 0, \;\; F^{lk}(0)\nabla_l F^{ij}(0) = F^{lj}(0)\nabla_l F^{ik}(0). \tag{1.3}$$

It is worth noting that the condition (1.3) is preserved by the flow, which has been proved in [12,34].

It is well-known that many fluids do not satisfy Newtonian law. There have been many attempts to capture different phenomena for non-Newtonian fluids; see the excellent survey paper [3,9,21,35] and references therein. The fluid of Oldroyd type is one of the classical non-Newtonian fluids with elastic property. Due to the physical importance and mathematical challenges, the studies on the equations of the viscoelastic flows of Oldroyd type have attracted many physicists and mathematicians during the last decade. In the direction of incompressible case, there are many important progresses on the investigation of the global existence and large time behavior of smooth solutions for sufficiently small initial, refer to [1,2,6,10,17,20–29,32,39] and references therein. The global existence of strong solution were proved by Lin et al. [27], Chen and Zhang [2], Lei et al. [20–24] in Hilbert space $H^\ell$, Qian [32] and Fang et al. [6,39] in critical Besov space for the Cauchy problem. The problem is much more complicated due to the lack of damping mechanism and boundary condition on $F$ for the initial boundary value problem. In order to bypass this difficult, Lin and Zhang [27] used some important relations, which was introduced by Sideris and Thomases [36], to show that some linear terms in the system are in fact high order terms and the global existence of strong solution was established. He and Xu [10] reformulated the original system along the particle trajectory and showed that in the Lagrangian coordinate, the system is decoupled and the well-posedness problem is reduced to the solvability of standard damped-wave equations with a no-slip boundary condition. They established the global existence and exponential decay estimate for the equivalent system with the small and smooth initial data. Recently, Hu and Lin [11] studied the problem for discontinuous initial data. Although there are some progress on the existence of weak solutions, the existence of weak solutions for arbitrary data is still an outstanding open question. In the direction of the compressible case, the local existence of multi-dimensional strong solution was obtained by Hu and Wang [12]. In [13,34], the global well-posedness in the critical Besov space with the lowest regularity was established. For the initial boundary value problem, global in time solution was proved to exist uniquely near the equilibrium state in [14,33].

In addition to global well-posedness, the problem of large time behavior is another important subject. Hu and Wu [15] provided a detailed analysis on the optimal time decay in $H^2$ framework. The whole system has been divided into two small systems which is similar to Navier–Stokes equations, which in turn facilitated the analysis, we refer the readers to [4,5,8,18,19,30,31] and references therein. Based on [16], Jia et al.