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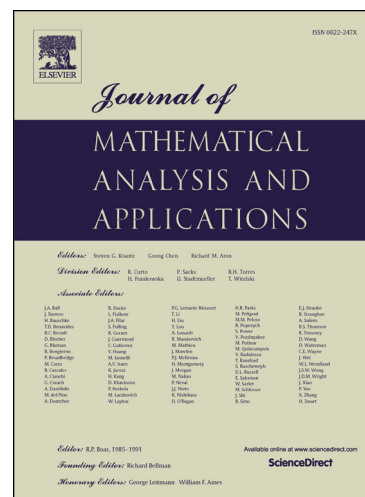
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SMOOTH COMPOSITIONS WITH A NONSMOOTH INNER FUNCTION

YONGJIE SHI AND CHENGJIE YU¹

ABSTRACT. Let $p : \mathbb{R} \rightarrow \mathbb{R}$ be a given function, and let \mathcal{A}_p be the set of smooth functions f such that $f(p(\cdot) + c)$ is smooth for any $c \in \mathbb{R}$. We show that if p is not smooth, then either every element of \mathcal{A}_p is constant, or there is a nonzero constant d such that \mathcal{A}_p equals to the set of smooth functions of periodicity d .

1. INTRODUCTION

In this paper, we prove the following result:

Theorem 1.1. *Let n be a nonnegative integer or ∞ , $p : \mathbb{R} \rightarrow \mathbb{R}$ be a given function with $p \notin C^n(\mathbb{R}, \mathbb{R})$ and*

$$(1.1) \quad \mathcal{A}_p = \{f \in C^n(\mathbb{R}, \mathbb{R}) \mid f(p(\cdot) + c) \in C^n(\mathbb{R}, \mathbb{R}) \text{ for any } c \in \mathbb{R}\}.$$

Then, either $\mathcal{A}_p = \mathbb{R}$ or $\mathcal{A}_p = C_d^n(\mathbb{R}, \mathbb{R})$ for some nonzero constant d . Here, $\mathcal{A}_p = \mathbb{R}$ means that every element in \mathcal{A}_p is constant, and $C_d^n(\mathbb{R}, \mathbb{R})$ means the collection of all functions in $C^n(\mathbb{R}, \mathbb{R})$ of periodicity d .

This result is motivated by the work [1] of Christensen and Wu on diffeological vector spaces. A weaker form of Theorem 1.1 is needed in [1]. The method in [1] dealing with the weaker form seems not applicable to prove Theorem 1.1.

It is clear that \mathcal{A}_p is a translation invariant subalgebra of $C^n(\mathbb{R}, \mathbb{R})$. As an example, take $p = \chi_E$ with E an arbitrary subset of \mathbb{R} ($E \neq \emptyset, \mathbb{R}$). Then, it is clear that $\cos(2\pi x), \sin(2\pi x) \in \mathcal{A}_p$. In fact, it is not hard to see that $\mathcal{A}_p = C_1^n(\mathbb{R}, \mathbb{R})$ in this specific example.

A key ingredient in the proof of Theorem 1.1 is the following result about the continuity of maps with σ -compact graph.

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