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The limit behavior of the Riemann solutions to the generalized Chaplygin gas equations with a source term ☆

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A B S T R A C T

The paper is concerned with the limit behavior of the Riemann solutions to the inhomogeneous generalized Chaplygin gas equations. The formation of delta shock waves and the vacuum states are identified and analyzed as the pressure vanishes. Unlike the homogeneous case, the Riemann solutions are no longer self-similar. As the pressure vanishes, the Riemann solutions to the generalized Chaplygin gas equations with a friction term converge to the Riemann solutions to the pressureless gas dynamics model with a body force.

1. Introduction

One-dimensional isentropic flow in the gas dynamics with a source term can be written as

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + P)_x = \beta\rho, \end{cases} \tag{1.1}$$

where β is a constant, ρ, u represent the density and the velocity respectively. $P = P(\rho, \varepsilon)$ is the scaled generalized Chaplygin gas pressure $P = \varepsilon p$ where $\varepsilon > 0$ and

$$p = -\frac{A}{\rho^\alpha},\ 0 < \alpha \le 1, \tag{1.2}$$

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where $A > 0$ is a constant. It is easy to see that $\lim_{\varepsilon\to 0} P(\rho, \varepsilon) = 0$. The model of the generalized Chaplygin gas explains the acceleration of the universe through an exotic equation of state (1.2) causing it acts like dark matter at high density and like dark energy at low density [17]. Thus, the generalized Chaplygin gas allows for an unification of dark energy and dark matter [1,2,15,24].

Sun [25] considered the inhomogeneous generalized Chaplygin gas equations (1.1)–(1.2) and obtained the non-self-similar Riemann solutions by introducing a new state variable

$$v(x,t) = u(x,t) - \beta t. \tag{1.3}$$

The new velocity (1.3) was introduced by Faccanoni and Mangeney [8] to study the Riemann problem of the shallow water equations.

When $\alpha = 1$, (1.2) is the equation of state of the Chaplygin gas. For the Chaplygin gas equations with a friction term, Shen [21] obtained the Riemann solutions. The Riemann problem with delta initial data and the vanishing pressure limit problem were considered in [9] and [10] respectively.

For the homogeneous generalized Chaplygin gas equations, $\beta = 0$ in (1.1)–(1.2), Wang [26] studied the Riemann problem. Sheng, Wang and Yin [23] studied the vanishing pressure limits of the Riemann solutions.

For the Chaplygin gas equations without a source term, there are many results, the readers are referred to [3,11,12,16,13,28,27].

The limit system of (1.1)–(1.2) as $\varepsilon \to 0$ formally becomes the pressureless gas dynamics model with a source term

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2)_x = \beta\rho. \end{cases} \tag{1.4}$$

We can also obtain system (1.4) by taking the constant pressure where the force is assumed to be the gravity with β being the gravity constant [6]. System (1.4) can describe the motion process of free particles sticking under collision in the low temperature and the information of large-scale structures in the universe [7,18]. Shen [20] considered both the Riemann problem and the Riemann problem with delta initial data of system (1.4).

Li [14] introduced the method of vanishing pressure limit to study the isothermal gases dynamics model. Chen and Liu [4] identified and analyzed the formation of delta shocks and vacuum states in Riemann solutions to the Euler equations for isentropic fluids. They made a further step later to generalize the results to the nonisentropic fluids [5]. For more results on vanishing pressure, we refer the readers to [22,19,23,29,30].

In this paper, we focus on the vanishing pressure limits of Riemann solutions to the inhomogeneous generalized Chaplygin gas equations (1.1)–(1.2). Unlike the homogeneous case, the Riemann solutions are no longer self-similar. Moreover, the generalized Chaplygin gas equations (1.1)–(1.2) differ from the Chaplygin gas equations. In the present case, the characteristic fields are genuinely nonlinear, while in the latter case, the characteristic fields are linearly degenerate.

Now, we give our main results.

Theorem 1.1. *When the parameter $\varepsilon \to 0$, Riemann solutions of system (1.1)–(1.2) converge to the Riemann solutions of system (1.4). There are three cases.*

(1) When $u_- > u_+$, the two shock wave solutions firstly converge to a delta shock solution as ε drops to a certain parameter value ε_2 which depends on the initial data, then as ε goes to zero, the delta shock wave converges to a delta shock solution of (1.4).

(2) When $u_- < u_+$, as $\varepsilon \to 0$, the two rarefaction wave solutions converge to two contact discontinuities connecting the states $(u_\pm + \beta t, \rho_\pm)$ with a vacuum state between them.