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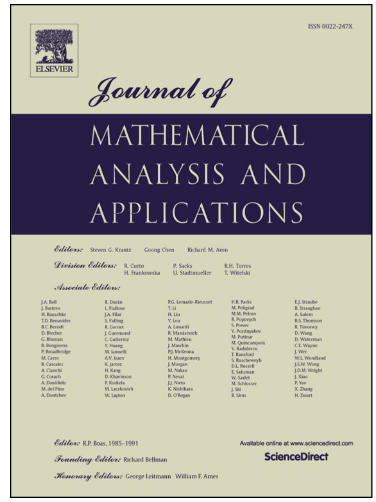

Ordered Probability Spaces

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ORDERED PROBABILITY SPACES

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Abstract. Let C be an open cone in a Banach space equipped with the Thompson metric with closure a normal cone. The main result gives sufficient conditions for Borel probability measures μ, ν on C with finite first moment for which $\mu \leq \nu$ in the stochastic order induced by the cone to be order approximated by sequences $\{\mu_n\}, \{\nu_n\}$ of uniform finitely supported measures in the sense that $\mu_n \leq \nu_n$ for each n and $\mu_n \to \mu$, $\nu_n \to \nu$ in the Wasserstein metric. This result is the crucial tool in developing a pathway for extending various inequalities on operator and matrix means, which include the harmonic, geometric, and arithmetic operator means on the cone of positive elements of a C^*-algebra, to the space $\mathcal{P}^1(C)$ of Borel measures of finite first moment on C. As an illustrative and important particular application, we obtain the monotonicity of the Karcher geometric mean on $\mathcal{P}^1(\mathbb{A}^+)$ for the positive cone $\mathbb{A}^+$ of a C^*-algebra $\mathbb{A}$.

1. Introduction

The set $\mathcal{P}^1(M)$ of Borel probability measures with finite first moment on a metric space M admits a standard metric called the Wasserstein metric. In [13] K.-T. Sturm considered contractive barycentric maps $\beta : \mathcal{P}^1(M) \to M$, maps that were metrically contractive and carried a point measure to the corresponding point. If M is complete, then a contractive barycentric map on the set $\mathcal{P}_0(M)$ of uniform, finitely supported probability measures extends uniquely to a barycentric map on $\mathcal{P}^1(M)$, since $\mathcal{P}_0(M)$ is dense in $\mathcal{P}^1(M)$ in the topology of the Wasserstein metric. In [8] Y. Lim and the author have modified this result by giving appropriate conditions for uniformity of an indexed family of symmetric means $\{G_n\}_{n\geq 1}$ to induce to a unique contractive barycentric map on $\mathcal{P}^1(M)$.

Key words and phrases. Borel probability measure, metric space, Wasserstein metric, barycentric map, partially ordered space.

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