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Ordered Probability Spaces

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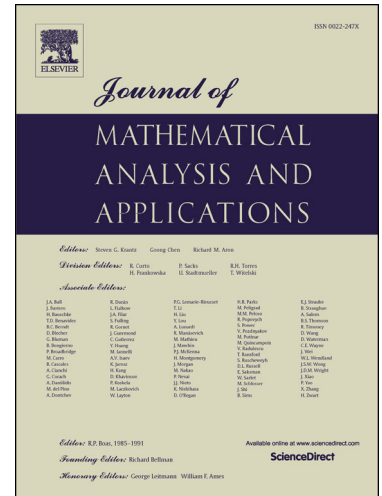
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## ORDERED PROBABILITY SPACES

JIMMIE LAWSON

ABSTRACT. Let  $C$  be an open cone in a Banach space equipped with the Thompson metric with closure a normal cone. The main result gives sufficient conditions for Borel probability measures  $\mu, \nu$  on  $C$  with finite first moment for which  $\mu \leq \nu$  in the stochastic order induced by the cone to be order approximated by sequences  $\{\mu_n\}, \{\nu_n\}$  of uniform finitely supported measures in the sense that  $\mu_n \leq \nu_n$  for each  $n$  and  $\mu_n \rightarrow \mu, \nu_n \rightarrow \nu$  in the Wasserstein metric. This result is the crucial tool in developing a pathway for extending various inequalities on operator and matrix means, which include the harmonic, geometric, and arithmetic operator means on the cone of positive elements of a  $C^*$ -algebra, to the space  $\mathcal{P}^1(C)$  of Borel measures of finite first moment on  $C$ . As an illustrative and important particular application, we obtain the monotonicity of the Karcher geometric mean on  $\mathcal{P}^1(\mathbb{A}^+)$  for the positive cone  $\mathbb{A}^+$  of a  $C^*$ -algebra  $\mathbb{A}$ .

## 1. INTRODUCTION

The set  $\mathcal{P}^1(M)$  of Borel probability measures with finite first moment on a metric space  $M$  admits a standard metric called the Wasserstein metric. In [13] K.-T. Sturm considered contractive barycentric maps  $\beta : \mathcal{P}^1(M) \rightarrow M$ , maps that were metrically contractive and carried a point measure to the corresponding point. If  $M$  is complete, then a contractive barycentric map on the set  $\mathcal{P}_0(M)$  of uniform, finitely supported probability measures extends uniquely to a barycentric map on  $\mathcal{P}^1(M)$ , since  $\mathcal{P}_0(M)$  is dense in  $\mathcal{P}^1(M)$  in the topology of the Wasserstein metric. In [8] Y. Lim and the author have modified this result by giving appropriate conditions for uniformity of an indexed family of symmetric means  $\{G_n\}_{n \geq 1}$  to induce to a unique contractive barycentric map on  $\mathcal{P}^1(M)$ .

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*Key words and phrases.* Borel probability measure, metric space, Wasserstein metric, barycentric map, partially ordered space.

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