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Rate of convergence for polymers in a weak disorder

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A B S T R A C T

We consider directed polymers in random environment on the lattice $\mathbb{Z}^d$ at small inverse temperature and dimension $d \geq 3$. Then, the normalized partition function W_n is a regular martingale with limit W. We prove that $n^{(d-2)/4}(W_n - W)/W_n$ converges in distribution to a Gaussian law. Both the polynomial rate of convergence and the scaling with the martingale W_n are different from those for polymers on trees.

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1. Polymer models and statement of the main result

1.1. Motivation

We consider directed polymers in random environment, given by a simple random walk on the d-dimensional lattice in a space–time random potential. In a seminal paper, Derrida and Spohn [12] perform a detailed analysis of polymers on the Cayley tree, or equivalently, the branching random walk with a fixed branching number. Later the same model has been taken up as an approximation and a toy model with explicit computations: in the physics literature, we mention the pleasant, recent and documented survey [15], and also [11] for the statistics of extremes on the hierarchical tree at zero temperature; on the mathematical side, the authors of [2] study the near-critical scaling window on the tree, the analogue of the intermediate disorder regime where the rescaled lattice model on line converges to the KPZ continuum random polymer [1,7]. Not only a source of inspiration and guidance, this model, as well as related random cascades, were also found to provide quantitative bounds on polymer models on the lattice in [9,26,27].

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In spite of these similarities, the two models behave quite differently in many aspects. In the strong disorder phase, the free energy of the branching process is linear in the inverse temperature β though it is strictly convex for the polymer on the lattice, see Theorem 1.5 in [8] in the case of a Bernoulli environment. Also, the fluctuations are expected to be of a completely different nature in the two models. In this paper, we consider the weak disorder regime, and we show that the martingale convergence takes place at a polynomial rate, whereas it is exponential in the corresponding supercritical Galton–Watson process [16,17].

More precisely, it is shown in [16,17] that, for a Galton–Watson process (Z_n) with $Z_0 = 1$, $m = EZ_1 > 1$ and $EZ_1^2 < \infty$, the renormalized population size $W_n = Z_n/m^n$ is a regular martingale with limit W such that

$$m^{n/2}(W - W_n) \to aW^{1/2}G \quad \text{in distribution} \tag{1}$$

and

$$m^{n/2}\,\frac{(W - W_n)}{W_n^{1/2}} \to aG \quad \text{in distribution,} \tag{2}$$

where $a^2 = \frac{\mathrm{Var} Z_1}{m^2 - m}$, G is a Gaussian $\mathcal{N}(0, 1)$ distributed random variable independent of W. Similarly, for branching random walks, the convergence of the Biggins martingale to its limit is exponentially fast [20,21] in the regular case. Recently the same question was studied for a branching process in a random environment [19,31], leading to similar conclusions.

In this paper, we consider random polymers on the lattice in a time–space dependent random medium, deep inside the weak disorder regime. Similar to the supercritical case of a branching process, weak disorder can be defined as the regime where the natural martingale is regular [5,22], or where the polymer is diffusive [10]. It holds in space dimension $d \geq 3$ [23] and at a temperature larger than some critical value which can be estimated by second moment and entropy considerations [4,6,18]. In Theorem 1.1 below, we prove that, at large temperature, the *speed of convergence is polynomial* but not exponential, and the limit scales with W or W_n instead of their square root as in (1) and (2). Precisely, we show a central limit theorem for the difference between the martingale and its limit: the ratio of the difference divided by $n^{-(d-2)/4}$ times the martingale is asymptotically normal.

In view of (1) and (2), this limit behavior has two remarkable and unexpected features. The slowdown in the rate of convergence (compared to the branching case) is due to space correlations coming from further intersections between paths on the lattice but not on the tree. Also the unusual linear scaling in the martingale can be understood as coming from fluctuations, and quadratic variations scale like the square of the martingale.

The result helps us for a deeper comprehension of the polymers model, and opens a way for further limit theorems about it.

1.2. Notations

- *The random walk:* $(\{S_n\}_{n\geq 0}, P_x)$ is a nearest neighbor, symmetric simple random walk on the d-dimensional integer lattice $\mathbb{Z}^d$ starting from x, $d \geq 3$. We let $P = P_0$ and we denote by $P[f] = \int f\,dP$ the expectation of f with respect to P.
- *The random environment:* $\eta = \{\eta(n, x) : n \in \mathbb{N},\ x \in \mathbb{Z}^d\}$ is an independent and identically distributed (i.i.d.) sequence of real random variables (r.v.'s), non-constant, such that,

$$\lambda(\beta) := \ln \mathbb{E}[\exp(\beta\eta(0, 0))] < \infty \quad \text{for all } \beta \in \mathbb{R},$$

where we denote by $\mathbb{E}$ the expectation over the environment. The corresponding probability measure will be denoted by $\mathbb{P}$.

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