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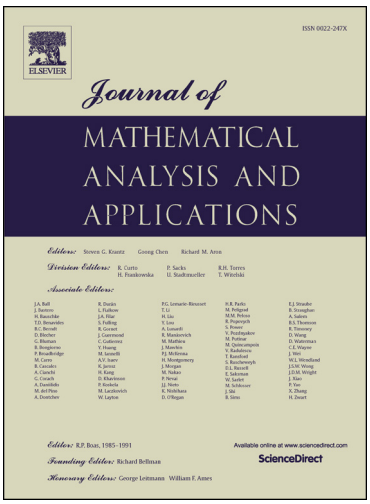

A parabolic–elliptic–elliptic attraction–repulsion chemotaxis system with logistic source

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A parabolic-elliptic-elliptic attraction-repulsion chemotaxis system with logistic source

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Abstract

This paper deals with the parabolic-elliptic-elliptic attraction-repulsion chemotaxis system with logistic source

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u) - \nabla \cdot (\chi u \nabla v) + \nabla \cdot (\xi u \nabla w) + ru - \mu u^2, & x \in \Omega, \ t > 0, \\ 0 = \Delta v + \alpha u - \beta v, & x \in \Omega, \ t > 0, \\ 0 = \Delta w + \gamma u - \delta w, & x \in \Omega, \ t > 0, \end{cases}$$

under no-flux boundary conditions in bounded domain with smooth boundary, where $\chi, \xi, \alpha, \beta, \gamma, \delta, r$ and μ are assumed to be positive.

When $\Omega \subseteq \mathbb{R}^3$, $D(u)$ is assumed to satisfy $D(0) > 0$, $D(u) \geq c_D u^{m-1}$ with $m \geq 1$ and $c_D > 0$, it is proved that if $\chi\alpha - \xi\gamma > 0$ and $\mu = \frac{1}{3}(\chi\alpha - \xi\gamma)$, then for any given $u_0 \in W^{1,\infty}(\Omega)$, the system possesses a global and bounded classical solution. For the case where $D(u) \equiv 1$ and $n \geq 3$, the convergence rate of the solution is established. When the random motion of the chemotactic species is neglected i.e. $(D(u) \equiv 0)$ and $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ is a convex domain, boundedness and the finite time blow up of the solution are investigated.

Keywords: Chemotaxis; Boundedness; Blow up; Asymptotic behavior; Logistic source

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1 Introduction

Chemotaxis is an interesting phenomenon which is used to measure the movement of cells in response to chemical substance. In 1970, Keller and Segel [8] (see also [7, 9]) introduced a model to describe the collective behavior of cells type which can be read as follows

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u) - \nabla \cdot (\chi u \nabla v) + f(u), & x \in \Omega, \ t > 0, \\ \tau v_t = \Delta v + u - v, & x \in \Omega, \ t > 0, \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0, & x \in \partial\Omega, \ t > 0, \\ u(x,0) = u_0(x), \ \ \tau v(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where $u(x,t)$ denotes the density of cells and $v(x,t)$ represents the concentration of the chemoattractant. The function $f : [0,\infty) \to \mathbb{R}$ is smooth and fulfills $f(0) > 0$. $\chi > 0$ is a parameter referred to as chemosensitivity.

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