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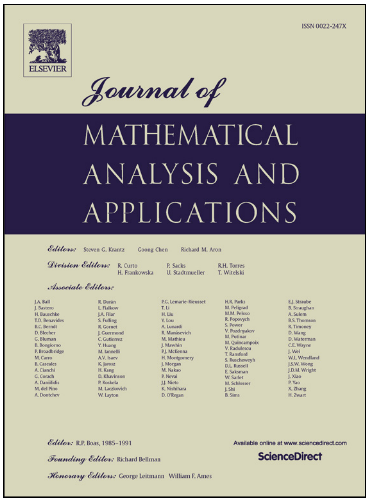

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LONG-TIME BEHAVIOR FOR A NONLOCAL CONVECTION DIFFUSION EQUATION

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Abstract. In this paper we consider a nonlocal viscous Burgers' equation and study the well-posedness and asymptotic behaviour of its solutions. We prove that under the smallness assumption on the initial data the solutions behave as the self similar profiles of the Burgers' equation with Dirac mass as the initial datum. The first term in the asymptotic expansion of the solutions is obtained by rescaling the solutions and proving the compactness of their trajectories.

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Key words: asymptotic behavior, nonlocal diffusion, nonlocal convection

1. Introduction

In this paper we analyze the following equation

$$\begin{cases} u_t(t,x) &= \displaystyle\int_{\mathbb{R}} K(x-y)(u(t,y)-u(t,x))dy \\ & \qquad\qquad + \displaystyle\int_{\mathbb{R}} G(x-y) f\Big(\frac{u(t,y)+u(t,x)}{2}\Big)dy, t>0, x\in\mathbb{R}, \\ u(0) &= \varphi, \end{cases} \tag{1.1}$$

in the particular case when $f(u)=u^2$. This model has been proposed in [4] as a regularization of the following nonlocal advection equation inspired by the peridynamic theory

$$u_t(t,x) = \int_{\mathbb{R}} G(x-y) f\Big(\frac{u(t,y)+u(t,x)}{2}\Big)dy.$$

The general model in [4] assumes that K is a nonnegative even function and G is an odd function. We consider here kernels K and G that are integrable. For simplicity we assume that K has mass one. We will analyze the well-posedness of system (1.1) and the long time behaviour of its solutions. The results presented here hold under the assumption that kernel K dominates G, i.e. for some positive constant $C=C_{GK}$ the following holds

$$|G(x)| \le C_{GK} K(x), \quad \forall\, x\in\mathbb{R}. \tag{1.2}$$

Using that function G is an odd function and the nonlinearity is given by $f(u)=u^2$ equation (1.1) can be written as

$$\begin{cases} u_t = K*u - u + G*\frac{u^2}{4} + (G*u)\frac{u}{2}, \quad t>0, x\in\mathbb{R}, \\ u(0)=\varphi. \end{cases} \tag{1.3}$$