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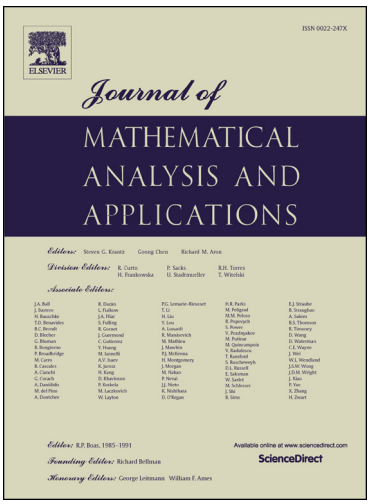

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Estimates for the first eigenvalue of the drifting Laplace and the p-Laplace operators on submanifolds with bounded mean curvature in the hyperbolic space

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Abstract

In this paper, we successfully give two interesting lower bounds for the first eigenvalue of submanifolds (with bounded mean curvature) in a hyperbolic space. More precisely, let M be an n-dimensional complete noncompact submanifold in a hyperbolic space and the norm of its mean curvature vector $\|H\|$ satisfies $\|H\| \leqslant \alpha < n-1$, then we prove that the first eigenvalue $\lambda_{1,p}(M)$ of the p-Laplacian Δ_p on M satisfies $\lambda_{1,p}(M) \geqslant \left(\frac{n-1-\alpha}{p}\right)^p$, $1 < p < \infty$, with equality achieved when M is totally geodesic and $p = 2$; let $(M, g, e^{-\varphi}dv_g)$ be an n-dimensional complete noncompact smooth metric measure space with M being a submanifold in a hyperbolic space, and $\|H\| \leqslant \alpha < n-1$, $\|\nabla\varphi\| \leqslant C$ with ∇ the gradient operator on M, then we show that the first eigenvalue $\lambda_{1,\varphi}(M)$ of the weighted Laplacian Δ_φ on M satisfies $\lambda_{1,\varphi}(M) \geqslant \frac{(n-1-\alpha-C)^2}{4}$, with equality attained when M is totally geodesic and $\varphi = constant$.

1 Introduction

Denote by $\mathscr{M}$ be an n-dimensional complete noncompact Riemannian manifold with the Laplace operator Δ. For an open bounded connected domain $\Omega \subseteq \mathscr{M}$, the classical Dirichlet eigenvalue problem is to find all possible real numbers λ such that there exists a nontrivial solution u to the boundary value problem

$$\begin{cases} \Delta u + \lambda u = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

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