

Accepted Manuscript

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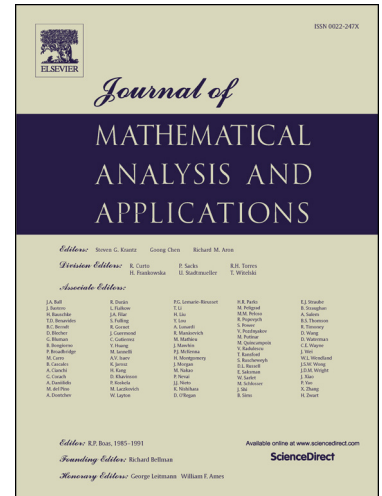
PII: S0022-247X(17)30706-0
DOI: <http://dx.doi.org/10.1016/j.jmaa.2017.07.043>
Reference: YJMAA 21574

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 29 December 2016

Please cite this article in press as: P. Li, S. Ponnusamy, Representation formula and bi-Lipschitz continuity of solutions to inhomogeneous biharmonic Dirichlet problems in the unit disk, *J. Math. Anal. Appl.* (2017), <http://dx.doi.org/10.1016/j.jmaa.2017.07.043>

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**REPRESENTATION FORMULA AND BI-LIPSCHITZ
CONTINUITY OF SOLUTIONS TO INHOMOGENEOUS
BIHARMONIC DIRICHLET PROBLEMS IN THE UNIT DISK**

PEIJIN LI AND SAMINATHAN PONNUSAMY

ABSTRACT. The aim of this paper is twofold. First, we establish the representation formula and the uniqueness of the solutions to a class of inhomogeneous biharmonic Dirichlet problems, and then prove the bi-Lipschitz continuity of the solutions.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

Let \mathbb{C} denote the complex plane. For $a \in \mathbb{C}$, let $\mathbb{D}(a, r) = \{z : |z - a| < r\}$, where $r > 0$, and $\mathbb{D}_r = \mathbb{D}(0, r)$. In particular, let $\mathbb{D} = \mathbb{D}_1$ and $\mathbb{T} = \partial\mathbb{D}$, the boundary of \mathbb{D} . For domains D and Ω be domains in \mathbb{C} , a function $p : D \rightarrow \Omega$ is said to be L_1 -Lipschitz (resp. L_2 -co-Lipschitz) if for all $z, w \in D$,

$$|p(z) - p(w)| \leq L_1|z - w| \quad (\text{resp. } |p(z) - p(w)| \geq L_2|z - w|)$$

for some positive constants L_1 and L_2 . We say that p is *bi-Lipschitz* if it is both Lipschitz and co-Lipschitz.

The main aim of this paper is to discuss the representation formula, the uniqueness and the bi-Lipschitz continuity of the solutions to the following inhomogeneous biharmonic Dirichlet problem (briefly, IBDP in the following):

$$(1.1) \quad \begin{cases} \Delta^2\Phi = g & \text{in } \mathbb{D}, \\ \Phi = f & \text{on } \mathbb{T}, \\ \partial_n\Phi = h & \text{on } \mathbb{T}, \end{cases}$$

where Δ denotes the Laplacian given by

$$\Delta = \frac{\partial^2}{\partial z \partial \bar{z}} = \frac{1}{4} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right),$$

∂_n denotes the differentiation in the inward normal direction and the boundary data f and $h \in \mathcal{D}'(\mathbb{T})$, the space of distributions in \mathbb{T} .

Note that a solution to the biharmonic equation $\Delta^2\Phi = 0$ is called a *biharmonic function*. See Almansi [6], Venkua [30] and [1, 2, 3, 13, 14] for properties of biharmonic functions.

2000 *Mathematics Subject Classification*. Primary: 31A30, 30C62; Secondary: 26A16, 34B27.

Key words and phrases. Biharmonic mapping, biharmonic operator, Dirichlet problem, Green function, Lipschitz continuity.

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