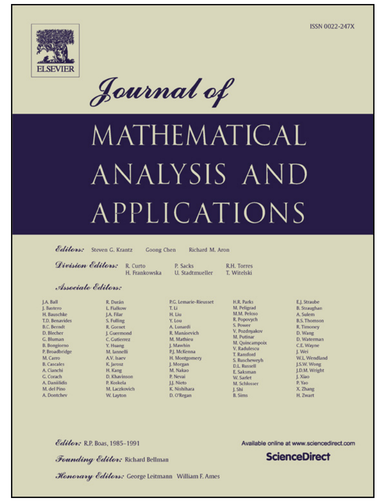
Accepted Manuscript



Representation formula and bi-Lipschitz continuity of solutions to inhomogeneous biharmonic Dirichlet problems in the unit disk

Peijin Li, Saminathan Ponnusamy

PII: S0022-247X(17)30706-0
DOI: http://dx.doi.org/10.1016/j.jmaa.2017.07.043
Reference: YJMAA 21574

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 29 December 2016

REPRESENTATION FORMULA AND BI-LIPSCHITZ CONTINUITY OF SOLUTIONS TO INHOMOGENEOUS BIHARMONIC DIRICHLET PROBLEMS IN THE UNIT DISK

PEIJIN LI AND SAMINATHAN PONNUSAMY

Abstract. The aim of this paper is twofold. First, we establish the representation formula and the uniqueness of the solutions to a class of inhomogeneous biharmonic Dirichlet problems, and then prove the bi-Lipschitz continuity of the solutions.

1. Introduction and statement of the main results

Let $\mathbb{C}$ denote the complex plane. For $a \in \mathbb{C}$, let $\mathbb{D}(a, r) = \{z : |z - a| < r\}$, where $r > 0$, and $\mathbb{D}_r = \mathbb{D}(0, r)$. In particular, let $\mathbb{D} = \mathbb{D}_1$ and $\mathbb{T} = \partial\mathbb{D}$, the boundary of $\mathbb{D}$. For domains D and Ω be domains in $\mathbb{C}$, a function $p : D \to \Omega$ is said to be L_1-*Lipschitz* (resp. L_2-*co-Lipschitz*) if for all $z, w \in D$,

$$|p(z) - p(w)| \leq L_1|z - w| \ \ (\text{resp. } |p(z) - p(w)| \geq L_2|z - w|)$$

for some positive constants L_1 and L_2. We say that p is *bi-Lipschitz* if it is both Lipschitz and co-Lipschitz.

The main aim of this paper is to discuss the representation formula, the uniqueness and the bi-Lipschitz continuity of the solutions to the following inhomogeneous biharmonic Dirichlet problem (briefly, IBDP in the following):

$$\begin{cases} \Delta^2\Phi = g & \text{in } \mathbb{D}, \\ \Phi = f & \text{on } \mathbb{T}, \\ \partial_n\Phi = h & \text{on } \mathbb{T}, \end{cases} \tag{1.1}$$

where Δ denotes the Laplacian given by

$$\Delta = \frac{\partial^2}{\partial z \partial \overline{z}} = \frac{1}{4}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right),$$

∂_n denotes the differentiation in the inward normal direction and the boundary data f and $h \in \mathfrak{D}'(\mathbb{T})$, the space of distributions in $\mathbb{T}$.

Note that a solution to the biharmonic equation $\Delta^2\Phi = 0$ is called a *biharmonic function*. See Almansi [6], Venkua [30] and [1, 2, 3, 13, 14] for properties of biharmonic functions.

2000 *Mathematics Subject Classification.* Primary: 31A30, 30C62; Secondary: 26A16, 34B27.

Key words and phrases. Biharmonic mapping, biharmonic operator, Dirichlet problem, Green function, Lipschitz continuity.