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Spectral analysis of certain groups of isometries on Hardy and Bergman spaces ☆

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ABSTRACT

Using the similarity theory of semigroups as well as spectral theory, we obtain the resolvents of the generators of strongly continuous groups of isometries on the Hardy and Bergman spaces. These groups are obtained as weighted composition operators associated with specific automorphisms of the upper half-plane. The resulting resolvents are given as integral operators for which we determine the norms and spectra.

1. Introduction

Let $\mathbb{C}$ be the complex plane. The set $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$, is called the (open) unit disc. Let dA denote the area measure on $\mathbb{D}$, and for $\alpha \in \mathbb{R}$, $\alpha > -1$, we define a positive Borel measure dm_α on $\mathbb{D}$ by $dm_\alpha(z) = (1 - |z|^2)^\alpha dA(z)$. On the other hand, the set $\mathbb{U} = \{\omega \in \mathbb{C} : \Im(\omega) > 0\}$ denotes the upper half of the complex plane $\mathbb{C}$, and where $\Im(\omega)$ stands for the imaginary part of ω. For $\alpha > -1$, we define a weighted measure on $\mathbb{U}$ by $d\mu_\alpha(\omega) = (\Im(\omega))^\alpha \, dA(\omega)$. The Cayley transform $\psi(z) := \frac{i(1+z)}{1-z}$ maps the unit disc $\mathbb{D}$ conformally onto the upper half-plane $\mathbb{U}$ with inverse $\psi^{-1}(\omega) = \frac{\omega - i}{\omega + i}$.

For an open subset Ω of $\mathbb{C}$, let $\mathcal{H}(\Omega)$ denote the Fréchet space of analytic functions $f : \Omega \to \mathbb{C}$ endowed with the topology of uniform convergence on compact subsets of Ω. Let $\mathrm{Aut}(\Omega) \subset \mathcal{H}(\Omega)$ denote the group of biholomorphic maps $f : \Omega \to \Omega$. For $1 \leq p < \infty$, the Hardy spaces of the upper half plane, $H^p(\mathbb{U})$, are defined as

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$$H^P(\mathbb{U}) := \left\{ f \in \mathcal{H}(\mathbb{U}) : \|f\|_{H^p(\mathbb{U})} := \sup_{y>0} \left(\int_{-\infty}^{\infty} |f(x+iy)|^p dx \right)^{1/p} < \infty \right\},$$

while the Hardy spaces of the unit disc, $H^p(\mathbb{D})$, by

$$H^P(\mathbb{D}) := \left\{ f \in \mathcal{H}(\mathbb{D}) : \|f\|^p_{H^p(\mathbb{D})} := \sup_{0<r<1} \frac{1}{2\pi} \int_{-\pi}^{\pi} |f(re^{i\theta})|^p \, d\theta < \infty \right\}.$$

We note that every function $f \in H^p(\mathbb{U})$ (or $H^p(\mathbb{D})$) has non-tangential boundary values almost everywhere on $\partial\mathbb{U}$ (or $\partial\mathbb{D}$), see for example [8]. In particular, H^p-functions may be identified with their boundary values and with this convention,

$$\|f\|_{H^p(\mathbb{U})} = \left(\int_{-\infty}^{\infty} |f(x)|^p \, dx \right)^{\frac{1}{p}} \text{ and } \|f\|_{H^p(\mathbb{D})} = \left(\int_0^{2\pi} |f(e^{i\theta})|^p \, d\theta \right)^{\frac{1}{p}}.$$

On the other hand, for $1 \le p < \infty$, $\alpha > -1$, the weighted Bergman spaces on the upper half plane, $L^p_a(\mathbb{U}, \mu_\alpha)$, are defined by

$$L^p_a(\mathbb{U}, \mu_\alpha) := \left\{ f \in \mathcal{H}(\mathbb{U}) : \|f\|_{L^p_a(\mathbb{U},\mu_\alpha)} = \left(\int_{\mathbb{U}} |f(z)|^p \, d\mu_\alpha(z) \right)^{\frac{1}{p}} < \infty \right\},$$

while the corresponding spaces on the disc, $L^p_a(\mathbb{D}, m_\alpha)$, by

$$L^p_a(\mathbb{D}, m_\alpha) := \left\{ f \in \mathcal{H}(\mathbb{D}) : \|f\|_{L^p_a(\mathbb{D},m_\alpha)} = \left(\int_{\mathbb{D}} |f(z)|^p \, dm_\alpha(z) \right)^{\frac{1}{p}} < \infty \right\}.$$

In particular, $L^p_a(\cdot) = L^P(\cdot) \cap \mathcal{H}(\cdot)$ where $L^p(\cdot)$ denotes the classical Lebesgue spaces. For a comprehensive theory of Hardy and Bergman spaces, we refer to [8,9,12,15,16]. As noted in [1] and [3], the Hardy space $H^p(\cdot)$ behaves in many ways as the limiting case of $L^p_a(\cdot)$ as $\alpha \to -1^+$. Therefore, we shall let X denote either the Hardy space $H^p(\mathbb{U})$ or the weighted Bergman space $L^p_a(\mathbb{U}, \mu_\alpha)$, and we associate with each X, a parameter $\gamma = \frac{\alpha+2}{p}$, where $\alpha = -1$ in the case that $X = H^p(\mathbb{U})$. Also, we shall let $X(\mathbb{D})$ denote the corresponding space of analytic functions on the unit disc $\mathbb{D}$.

If X is an arbitrary Banach space, let $\mathcal{L}(X)$ denote the algebra of bounded linear operators on X. For a linear operator T with domain $\mathcal{D}(T) \subset X$, denote the spectrum and point spectrum of T by $\sigma(T, X)$ and $\sigma_p(T, X)$ respectively. The resolvent set of T is $\rho(T, X) = \mathbb{C} \setminus \sigma(T, X)$ while $r(T)$ denotes its spectral radius. For a good account of the theory of spectra, see [6,7,13]. If X and Y are arbitrary Banach spaces and $U \in \mathcal{L}(X, Y)$ is an invertible operator, then clearly $(A_t)_{t\in\mathbb{R}} \subset \mathcal{L}(X)$ is a strongly continuous group if and only if $B_t := UA_tU^{-1}$, $t \in \mathbb{R}$, is a strongly continuous group in $\mathcal{L}(Y)$. In this case, if $(A_t)_{t\in\mathbb{R}}$ has generator Γ, then $(B_t)_{t\in\mathbb{R}}$ has generator $\Delta = U\Gamma U^{-1}$ with domain $\mathcal{D}(\Delta) = U\mathcal{D}(\Gamma) := \{y \in Y : Uy \in \mathcal{D}(\Gamma)\}$. Moreover, $\sigma_p(\Delta, Y) = \sigma_p(\Gamma, X)$, and $\sigma(\Delta, Y) = \sigma(\Gamma, X)$, since if λ is in the resolvent set $\rho(\Gamma, X) := \mathbb{C} \setminus \sigma(\Gamma, X)$, we have that $R(\lambda, \Delta) = UR(\lambda, \Gamma)U^{-1}$. See for example [10, Chapter II] and [13, Chapter 3].

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