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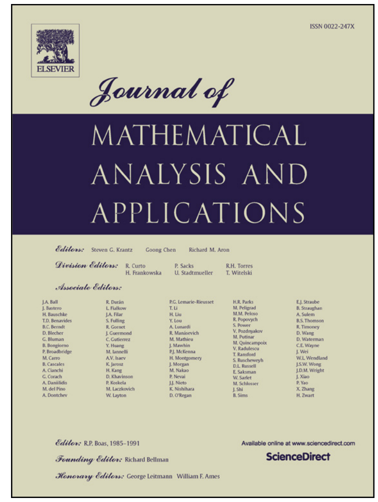

Lower dimensions of some fractal sets

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Lower dimensions of some fractal sets*

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Abstract

In this paper, we obtain the lower dimension formula of compact subsets in the doubling metric space. Using this result, we give the lower dimension formula of the homogeneous sets. We also study the lower dimensions of Moran sets under the suitable condition.

Keywords Lower dimension, Moran sets, Doubling metric space.
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1 Introduction

The lower dimension was first introduced by Larman[19], and it has been referred to by other equivalent definitions, for example, lower Assouad dimension by Käenmäki et al.[17], etc. Let (X, d) be a metric space, for any bounded subset $E \subset X$, denote $N_r(E)$ as the smallest number of balls of radius r needed to cover E. For any set $E \subset X$, denote $N_{r,R}(E)$ as $\inf_{x\in E} N_r(B(x, R) \cap E)$, where $B(x, R)$ is an open ball centered on x with radius R. The lower dimension of the set $E \subset X$ is defined as

$$\dim_L E = \sup\{s \geq 0| \text{ There exist } b, c > 0, \text{ such that for any } 0 < r < R < b, \text{ we have } N_{r,R}(E) \geq c(\frac{R}{r})^s\}.$$

The lower dimension is a natural dual to the Assouad dimension[1, 2], and these two dimensions often come in pairs. They are essential tools for studying the homogeneity of fractal sets. It is well-known that the lower dimension and Assouad dimension of self-similar sets with open set condition are equal to the similarity dimension, but they are not equal in general. It is well-known that if $E \subset X$ is a totally bounded subset, then

$$\dim_L E \leq \underline{\dim}_B E \leq \overline{\dim}_B E \leq \dim_A E,$$

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