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Abstract

We use the concept of reproducing pairs to study Gabor systems at critical density. First, we present a generalization of the Balian-Low theorem to the reproducing pairs setting. Then, we prove our main result that there exists a reproducing partner for the Gabor system of integer time-frequency shifts of the Gaussian. In other words, the coefficients for this Gabor expansion of a square integrable function can be calculated using inner products with an unstructured family of vectors in $L^2(\mathbb{R})$. This solves one of the last few open questions for this system.

1. Introduction

The main problem of Gabor analysis is to understand the conditions and obstructions on the family $G(g, \Lambda) := \{T_{\lambda_1} M_{\lambda_2} g\}_{\lambda \in \Lambda} \subset L^2(\mathbb{R})$ to be a frame. There exists, however, a great abundance of windows g and lattices Λ generating Gabor families which are, on the one hand, complete and, on the other hand, violate at least one of the frame bounds. The well-known Balian-Low theorem, for example, states that the window function of a Gabor frame at the

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