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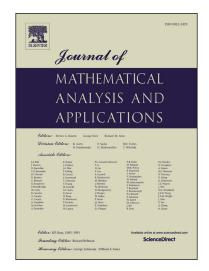
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### **ACCEPTED MANUSCRIPT**

# ON THE ANALYTIC SMOOTHING EFFECT FOR THE HARTREE EQUATION WITH A SHORT RANGE INTERACTION POTENTIAL

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ABSTRACT. We consider the Cauchy problem for the Hartree equation in space dimension  $d \geq 2$ . We assume that the interaction potential V(x) is short range. More precisely, we consider the case where V belongs to the weak  $L^{d/\sigma}$  space with  $1 < \sigma < d$ . We prove that if  $2 \leq \sigma < d$  (resp.  $1 < \sigma < 2$ ), the initial data  $\phi$  is small in the sense of the homogeneous Sobolev space  $\dot{H}^{\sigma/2-1}$  (resp. the homogeneous weighted Sobolev space  $\mathcal{F}\dot{H}^{1-\sigma/2}$ ) and the Fourier transform  $\mathcal{F}\phi$  satisfies a real-analytic condition, then the corresponding solution u(t) is also real-analytic for any  $t \neq 0$ . We remark that no  $\dot{H}^{\sigma/2-1}$  (resp.  $\mathcal{F}\dot{H}^{1-\sigma/2}$ ) smallness condition is imposed on first and higher order partial derivatives of  $\mathcal{F}\phi$  when  $2 \leq \sigma < d$  (resp.  $1 < \sigma < 2$ ).

#### 1. Introduction

We consider the Cauchy problem for the nonlinear Schrödinger equation of the form

$$\begin{cases} iu_t + \Delta u = F(u), \\ u(0, x) = \phi(x). \end{cases}$$
 (1.1)

Here, u is a complex-valued unknown function of  $(t,x) \in \mathbb{R} \times \mathbb{R}^d$ ,  $d \geq 2$ ,  $i = \sqrt{-1}$ ,  $\Delta$  is the Laplacian in  $\mathbb{R}^d$ , F(u) denotes the Hartree term  $(V*|u|^2)u$  and \* is the convolution in  $\mathbb{R}^d$ . Throughout this paper, we assume that the interaction potential V is a complex-valued given function on  $\mathbb{R}^d$  and belongs to the weak  $L^{d/\sigma}$  space with  $1 < \sigma < d$ . In other words, we assume that  $1 < \sigma < d$  and

$$\sup_{\lambda>0} \lambda \,\mu\Big(\left\{x \in \mathbb{R}^d; |V(x)| > \lambda\right\}\Big)^{\sigma/d} < \infty,\tag{1.2}$$

where  $\mu$  is the Lebesgue measure on  $\mathbb{R}^d$ . There is a large literature on the Cauchy problem for nonlinear Schrödinger equations (see, e.g., [2, 19, 35] and references therein). In this paper, we consider the analytic smoothing effect of time-global solutions to (1.1).

Before considering the analytic smoothing effect, we first state the existence and asymptotics of time-global solutions to (1.1). For this purpose, we list some notation. For  $1 \leq r \leq \infty$ , we denote the Lebesgue space  $L^r(\mathbb{R}^d)$  and its norm by  $L^r$  and  $\|\cdot\|_r$ , respectively. We set  $\omega_0 = \sqrt{-\Delta}$ . For  $1 < r < \infty$  and  $0 \leq s < d/r$ , we define the homogeneous

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Key words and phrases. Hartree equation; analytic smoothing effect; short range potential.

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