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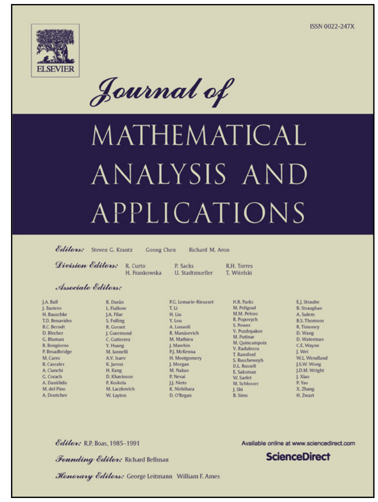

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ON THE ANALYTIC SMOOTHING EFFECT FOR THE HARTREE EQUATION WITH A SHORT RANGE INTERACTION POTENTIAL

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Abstract. We consider the Cauchy problem for the Hartree equation in space dimension $d \geq 2$. We assume that the interaction potential $V(x)$ is short range. More precisely, we consider the case where V belongs to the weak $L^{d/\sigma}$ space with $1 < \sigma < d$. We prove that if $2 \leq \sigma < d$ (resp. $1 < \sigma < 2$), the initial data ϕ is small in the sense of the homogeneous Sobolev space $\dot{H}^{\sigma/2-1}$ (resp. the homogeneous weighted Sobolev space $\mathcal{F}\dot{H}^{1-\sigma/2}$) and the Fourier transform $\mathcal{F}\phi$ satisfies a real-analytic condition, then the corresponding solution $u(t)$ is also real-analytic for any $t \neq 0$. We remark that no $\dot{H}^{\sigma/2-1}$ (resp. $\mathcal{F}\dot{H}^{1-\sigma/2}$) smallness condition is imposed on first and higher order partial derivatives of $\mathcal{F}\phi$ when $2 \leq \sigma < d$ (resp. $1 < \sigma < 2$).

1. Introduction

We consider the Cauchy problem for the nonlinear Schrödinger equation of the form

$$\begin{cases} iu_t + \Delta u = F(u), \\ u(0,x) = \phi(x). \end{cases} \tag{1.1}$$

Here, u is a complex-valued unknown function of $(t,x) \in \mathbb{R} \times \mathbb{R}^d$, $d \geq 2$, $i = \sqrt{-1}$, Δ is the Laplacian in $\mathbb{R}^d$, $F(u)$ denotes the Hartree term $(V * |u|^2)u$ and $*$ is the convolution in $\mathbb{R}^d$. Throughout this paper, we assume that the interaction potential V is a complex-valued given function on $\mathbb{R}^d$ and belongs to the weak $L^{d/\sigma}$ space with $1 < \sigma < d$. In other words, we assume that $1 < \sigma < d$ and

$$\sup_{\lambda>0} \lambda\, \mu\Big(\big\{ x \in \mathbb{R}^d; |V(x)| > \lambda \big\} \Big)^{\sigma/d} < \infty, \tag{1.2}$$

where μ is the Lebesgue measure on $\mathbb{R}^d$. There is a large literature on the Cauchy problem for nonlinear Schrödinger equations (see, e.g., [2, 19, 35] and references therein). In this paper, we consider the analytic smoothing effect of time-global solutions to (1.1).

Before considering the analytic smoothing effect, we first state the existence and asymptotics of time-global solutions to (1.1). For this purpose, we list some notation. For $1 \leq r \leq \infty$, we denote the Lebesgue space $L^r(\mathbb{R}^d)$ and its norm by L^r and $\|\cdot\|_r$, respectively. We set $\omega_0 = \sqrt{-\Delta}$. For $1 < r < \infty$ and $0 \leq s < d/r$, we define the homogeneous

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