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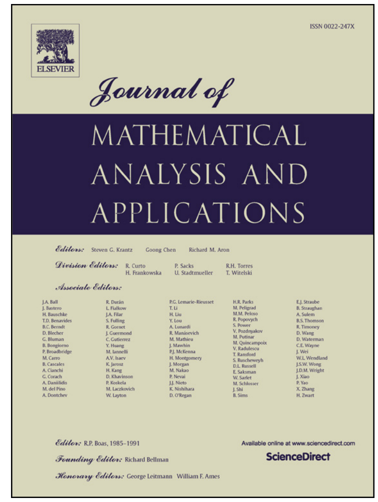

On the planar Brownian Green's function for stopping times

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On the planar Brownian Green's function for stopping times.

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Abstract

It has been known for some time that the Green's function of a planar domain can be defined in terms of the exit time of Brownian motion, and this definition has been extended to stopping times more general than exit times. In this paper, we extend the notion of conformal invariance of Green's function to analytic functions which are not injective, and use this extension to calculate the Green's function for a stopping time defined by the winding of Brownian motion. These considerations lead to a new proof of the Riemann mapping theorem. We also show how this invariance can be used to deduce several identities, including the standard infinite product representations of several trigonometric functions.

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1 Introduction

In the field of analysis, Green's function $G(x, y)$ on regions of $\mathbb{R}^n$ is formally defined to be the solution of $LG(x, y) = \delta(y - x)$, where L is a linear differential operator. In complex analysis, where the Laplacian is the differential operator of most importance, for a given domain $\Omega \subseteq \mathbb{C}$ and $z \in \Omega$ the Green's function of the Laplacian is generally defined by the following.

Definition 1. *The Green's function $G_\Omega(z, w)$ on a domain Ω is a function in w on $\Omega \backslash \{z\}$ satisfying the following properties.*