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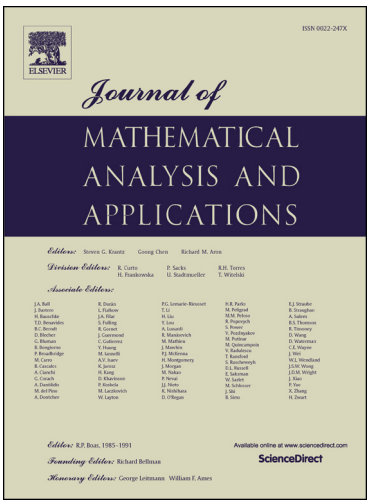

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ATOMIC DECOMPOSITION OF HARDY-AMALGAM SPACES

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Abstract. We define a Hardy type space, by taking in the maximal characterization of Hardy spaces, the Wiener amalgam norms of the maximal functions, instead of the Lebesgue norms. The functions in this space can then behave differently locally and at infinity. We prove that this space contains the classical Hardy space and obtain an atomic decomposition.

1. Introduction

The seminal work of Stein and Weiss [27], was the starting point of various generalizations of Hardy spaces. Let $\varphi \in \mathcal{C}^\infty(\mathbb{R}^d)$ with support on $B(0,1)$ such that $\int \varphi dx = 1$, where $B(0,1)$ is the unit open ball centered at 0 and $\mathcal{C}^\infty(\mathbb{R}^d)$ denotes the space of infinitely differentiable complex values functions on $\mathbb{R}^d$. For $t > 0$, we denote by φ_t the dilated function $\varphi_t(x) = t^{-d}\varphi(x/t)$. The Hardy space $\mathcal{H}^p(\mathbb{R}^d)$ is defined as the space of tempered distributions f such that the function

$$\mathcal{M}(f) := \sup_{t>0} |f * \varphi_t| \tag{1.1}$$

is in $L^p(\mathbb{R}^d)$. It is well-known (see [8]), not only that the definition does not depend on the particular function φ, but that the Hardy space $\mathcal{H}^p(\mathbb{R}^d)$ can be characterized in terms of grand maximal function.

Several generalizations of these $\mathcal{H}^p(\mathbb{R}^d)$ spaces have been considered, such as Hardy spaces on weighted $L^p(\mathbb{R}^d)$ spaces (see [13]) generalized in the case of Musielak-Orlicz spaces by Ky [18], Hardy spaces $H^{p,q}(\mathbb{R}^d)$ modeled on classical Lorentz spaces $L^{p,q}(\mathbb{R}^d)$ (see [1]) and Hardy spaces $H^q(\phi)$ modeled on generalized Lorentz spaces $\Lambda_q(\phi)$ (see [2]). In [30], Weisz defined Hardy type space $\mathcal{H}^1_{\text{weak}}$ which consists of measurable functions f such that $\mathcal{M}(f)$ belongs to the weak Lebesgue space $L^{1,\infty}(\mathbb{R}^d)$, spaces generalized in [21] with the weak Orlicz space replacing weak Lebesgue.

Following the maximal function approach, we study Hardy-amalgam spaces $\mathcal{H}^{(p,q)}(\mathbb{R}^d)$ by taking the Wiener amalgam "norm" in the definition of the Hardy space, instead of the Lebesgue "norm".

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