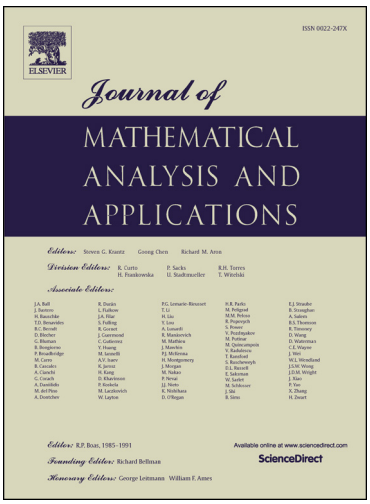
Accepted Manuscript



A concrete example with multiple limit cycles for three dimensional Lotka–Volterra systems

Kouichi Murakami

PII: S0022-247X(17)30748-5
DOI: http://dx.doi.org/10.1016/j.jmaa.2017.07.076
Reference: YJMAA 21607

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 26 February 2017

Please cite this article in press as: K. Murakami, A concrete example with multiple limit cycles for three dimensional Lotka–Volterra systems, *J. Math. Anal. Appl.* (2017), http://dx.doi.org/10.1016/j.jmaa.2017.07.076

A Concrete Example with Multiple Limit Cycles for Three Dimensional Lotka–Volterra Systems

Kouichi Murakami

Department of Mathematical Sciences, Faculty of Sciences and Technology, Tokushima University, Tokushima 770-8506, Japan

Abstract

The number of limit cycles for three dimensional Lotka–Volterra systems is an open problem. Recently, Yu et al. (2016) constructed some examples with the possibility of the existence of four limit cycles. Unfortunately, multiple limit cycles are not visible by numerical simulations, because all of them are very close to the interior equilibrium and extremely small. We present a concrete example with multiple limit cycles for three dimensional Lotka–Volterra systems which we can confirm them by numerical simulations. First we prepare the modified formula to compute coefficients of the normal form for the generalized Hopf bifurcation. Applying this formula to three dimensional Lotka–Volterra competitive systems with the aid of the computer algebra system, we derive the critical parameter values explicitly such that the interior equilibrium is exactly an unstable weak focus. Also we show that the heteroclinic cycle on the boundary of $\mathbb{R}^3_+$ is repelling. This implies that there exists a stable limit cycle by the Poincare–Bendixson theorem. Then, adding some suitable perturbations to parameters, we generate additional two limit cycles near the interior equilibrium by the generalized Hopf bifurcation. Finally we confirm that there exist three limit cycles by numerical simulations.

Keywords: 3D Lotka–Volterra competitive systems, multiple limit cycles, Hopf bifurcation

Email address: murakami.kouichi@tokushima-u.ac.jp (Kouichi Murakami)