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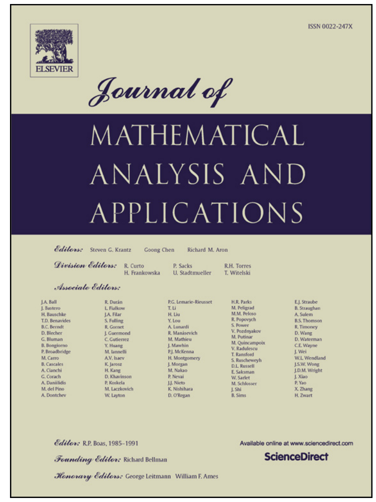


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# Spin actions in Euclidean and Hermitian Clifford analysis in superspace

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**Abstract**

In [4] we studied the group invariance of the inner product of supervectors as introduced in the framework of Clifford analysis in superspace. The fundamental group $\mathrm{SO}_0$ leaving invariant such an inner product turns out to be an extension of $\mathrm{SO}(m) \times \mathrm{Sp}(2n)$ and gives rise to the definition of the spin group in superspace through the exponential of the so-called extended superbivectors, where the spin group can be seen as a double covering of $\mathrm{SO}_0$ by means of the representation $h(s)[x] = sx\overline{s}$. In the present paper, we study the invariance of the Dirac operator in superspace under the classical $H$ and $L$ actions of the spin group on superfunctions. In addition, we consider the Hermitian Clifford setting in superspace, where we study the group invariance of the Hermitian inner product of supervectors introduced in [3]. The group of complex supermatrices leaving this inner product invariant constitutes an extension of $\mathrm{U}(m) \times \mathrm{U}(n)$ and is isomorphic to the subset $\mathrm{SO}_0^J$ of $\mathrm{SO}_0$ of elements that commute with the complex structure $J$. The realization of $\mathrm{SO}_0^J$ within the spin group is studied together with the invariance under its actions of the super Hermitian Dirac system. It is interesting to note that the spin element leading to the complex structure can be expressed in terms of the $n$-dimensional Fourier transform.

**Keywords.** Group invariance, Hermitian Clifford analysis, superspace, Dirac operator
**Mathematics Subject Classification (2010).** 30G35

## 1 Introduction

In a previous paper, [4], we have introduced the spin group $\mathrm{Spin}(m|2n)(\Lambda_N)$ in the framework of Clifford analysis in superspace, where we consider $m$ bosonic dimensions, $2n$ fermionic dimensions and the Grassmann algebra $\mathbb{R}\Lambda_N$ as the set of coefficients. Such a definition aims, as is the case in the classical setting, at describing the set of rotations in the space $\mathbb{R}^{m|2n}(\mathcal{V})$ of supervectors in terms of Clifford multiplication. However, here a more complicated situation arises since *supervector reflections* do not suffice for describing the whole set of supermatrices leaving the inner product of supervectors invariant. This is due to the structure of the real projection of the group $\mathrm{SO}_0(m|2n)(\mathbb{R}\Lambda_N)$ ($\mathrm{SO}_0$ for short) of superrotations, which is given by $\mathrm{SO}(m) \times \mathrm{Sp}(2n)$ and clearly contains a real, non-nilpotent symplectic part. The non-nilpotent part of the supervector variables is given only by the classical Clifford vector part. Therefore the reflections generated by the supervectors only include the real rotation group $\mathrm{SO}(m)$.

This issue may be solved by means of the extension $\mathbb{R}^{(2)E}_{m|2n}(\Lambda_N)$ of the Lie algebra of superbivectors, which turns out to be isomorphic to the Lie algebra $\mathfrak{so}_0(m|2n)(\mathbb{R}\Lambda_N)$ ($\mathfrak{so}_0$ for short) of $\mathrm{SO}_0$. The Lie group $\mathrm{SO}_0$ is connected, whence it can be fully described through finite products of exponentials of $\mathfrak{so}_0$–elements. Hence, the exponentials of the extended superbivectors in $\mathbb{R}^{(2)E}_{m|2n}(\Lambda_N)$ can be seen as generators of the spin group in this setting. In fact, such a group fully covers $\mathrm{SO}_0$ through the representation $h(s)[x] = sx\overline{s}$, $s \in \mathrm{Spin}(m|2n)(\Lambda_N)$, $x \in \mathbb{R}^{m|2n}(\mathcal{V})$. In particular, $\mathrm{SO}_0$ can be decomposed as the product of three exponential maps acting in some specific subspaces $\mathfrak{s}_1$, $\mathfrak{s}_2$, $\mathfrak{s}_3$ of $\mathfrak{so}_0$ where $\mathfrak{so}_0 = \mathfrak{s}_1 \oplus \mathfrak{s}_2 \oplus \mathfrak{s}_3$. The corresponding isomorphic decomposition for $\mathbb{R}^{(2)E}_{m|2n}(\Lambda_N)$ leads to a subset $S$ of $\mathrm{Spin}(m|2n)(\Lambda_N)$ which