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Margarida Baía, Farid Bozorgnia, Léonard Monsaingeon, Juha Videman

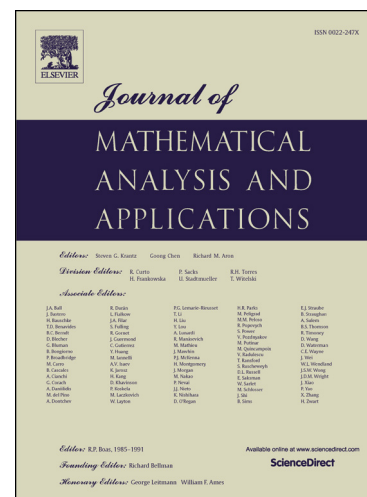
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A DEGENERATE ELLIPTIC-PARABOLIC SYSTEM ARISING IN COMPETITIVE CONTAMINANT TRANSPORT

MARGARIDA BAÍA, FARID BOZORGNIA, LÉONARD MONSAINGEON, AND JUHA VIDEMAN

ABSTRACT. The objective of this work is to study a coupled system of degenerate and nonlinear partial differential equations governing the transport of reactive solutes in groundwater. We show that this system admits a unique weak solution provided the nonlinear adsorption isotherm associated with the reaction process satisfies certain physically reasonable structural conditions, by addressing a more general problem. In addition, we conclude, that the solute concentrations stay non-negative if the source term is componentwise non-negative and investigate numerically the finite speed of propagation of compactly supported initial concentrations, in a two-component test case.

1. INTRODUCTION

Contaminant transport in groundwater and subsurface environments is a complex process given the combination of phenomena, phases and interfaces present in such environments. The transport can be influenced by an array of mechanical and chemical interactions, between or among the different constituent phases and species, such as advection, diffusion, dispersion and sorption, cf. [6, 7].

In this work, we study a system of partial differential equations inspired by multi-component contaminant transport models where the transport is dominated by adsorption and diffusion and where the adsorption process, adherence of a pollutant on the solid matrix at the fluid-solid interface, is assumed to be in equilibrium and is of Freundlich type, cf. Section 2 for a description of such models. Namely, our objective is to study the initial-boundary value problem

$$(1.1) \quad \begin{cases} \partial_t b(u) - \Delta u = f & \text{in } (0, T) \times \Omega, \\ u = u_b & \text{on } (0, T) \times \Gamma, \\ b(u|_{t=0}) = b_0 & \text{in } \Omega, \end{cases}$$

for the unknown solute concentrations $u : [0, T] \times \Omega \rightarrow \mathbb{R}^m$, where $f : [0, T] \times \Omega \rightarrow \mathbb{R}^m$, $b_0 : \Omega \rightarrow \mathbb{R}^m$ and $u_b : [0, T] \times \Gamma \rightarrow \mathbb{R}^m$ are given, $\Omega \subset \mathbb{R}^d$ is a smooth domain where the transport process takes place, $\Gamma = \partial\Omega$, $T > 0$, $m \geq 2$ and the vector field $b : \mathbb{R}^m \rightarrow \mathbb{R}^m$, associated with a Freundlich-type adsorption isotherm, is given by $b(u) = u + |u|^{p-1}u$, $p \in (0, 1)$. We note that $D_u b(u) \rightarrow \infty$ when $u \rightarrow 0$, i.e., the elliptic-parabolic system (1.1) becomes singular at $u = 0$. Solutions of system (1.1) may thus exhibit finite speed of propagation of compactly supported initial concentrations giving rise to free boundaries that separate the regions where the solute concentrations vanish from those where they are positive.

Given that $b = \nabla\phi$, for some $\phi : \mathbb{R}^m \rightarrow \mathbb{R}$ (cf. (2.7)), we may assume, more generally, that the vector field $b : \mathbb{R}^m \rightarrow \mathbb{R}^m$ in (1.1) can be expressed as a gradient of some non-negative C^1 -convex function $\phi : \mathbb{R}^m \rightarrow \mathbb{R}$. Under this assumption, problem (1.1) falls under the general quasi-linear elliptic-parabolic systems addressed by Alt and Luckhaus in their seminal paper [2]. The main difference between our approach and that of [2] lies in the existence proof which we base on Rothe's method ([23]) and on solving a convex minimization problem at each time step. We wish to emphasize that even though we are largely influenced by the results of [2], our approach provides a simple method for the numerical approximation of solutions to system (1.1), see Section 6.

If, for general systems of the form (1.1), the nonlinear term b is invertible as a function from \mathbb{R}^m to \mathbb{R}^m , one can make a change of variables and pass the degeneracy to the diffusion term. In that case, system (1.1) reduces to a coupled system of nonlinear diffusion equations of porous medium type or, in the one-component case, to a generalized porous medium equation. This inversion is possible for most adsorption isotherms,

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