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General decay result for nonlinear viscoelastic equations

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A B S T R A C T

In this paper we consider a nonlinear viscoelastic equation with minimal conditions on the $L^1(0,\infty)$ relaxation function g namely $g'(t) \le -\xi(t)H(g(t))$, where H is an increasing and convex function near the origin and ξ is a nonincreasing function. With only these very general assumptions on the behavior of g at infinity, we establish optimal explicit and general energy decay results from which we can recover the optimal exponential and polynomial rates when $H(s) = s^p$ and p covers the full admissible range $[1, 2)$. We get the best decay rates expected under this level of generality and our new results substantially improve several earlier related results in the literature.

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1. Introduction

In this paper we are concerned with the following nonlinear viscoelastic problem

$$\begin{cases} |u_t|^\rho u_{tt} - \Delta u - \Delta u_{tt} + \int_0^t g(t-s)\Delta u(s)ds = u\,|u|^\gamma, \text{ in } \Omega \times (0,\infty) \\ u = 0, \text{ on } \partial\Omega \times (0,\infty) \\ u(x,0) = u_0(x),\; u_t(x,0) = u_1(x), \quad x \in \Omega. \end{cases} \tag{1.1}$$

Here Ω is a bounded domain of $\mathbb{R}^n$ $(n \ge 1)$ with a smooth boundary $\partial\Omega$, $\min\{\rho,\gamma\} > 0$ and $(n-2)\max\{\rho,\gamma\} \le 2$, the integral term is a finite memory responsible for the viscoelastic damping where g is a positive decreasing function called the relaxation function, and the right hand side of $(1.1)_1$ is a source term. This type of problems arise in viscoelasticity and has been widely studied in the literature. The global existence was established for all cases, whether $\rho > 0$ or $\rho = 0$, and in the absence of the source term or in the presence of the source term provided the initial data lies in a stable set.

Over forty years ago, it was proved by Dafermos [11,12] that, for smooth monotone functions g, the solutions of viscoelastic equations go to zero as t goes to infinity. However, no rate of decay has been

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specified. Recently, Jin et al. [16] showed that the energy decays at least at the rate of $\frac{1}{t}$. For specific behavior of the relaxation function g, we refer to [4,5,15,24,25,28,34] for subsequent results showing that the energy decays exponentially (resp. polynomially) if g decays exponentially (resp. polynomially). The same results were obtained by Alabau-Boussouira et al. [2], Cannarsa et al. [6] and Rivera et al. [26,27] for more general abstract equations and by Cavalcanti et al. [8] for equations with both viscoelastic and frictional damping terms.

Then, a natural question was raised: how does the energy behave as the kernel function does not necessarily decay exponentially or polynomially? Messaoudi [22,23] looked at

$$u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau = b|u|^\gamma u,$$

for $b = 0$ or 1 and g satisfying

$$g'(t) \leq -\xi(t)g(t) \tag{1.2}$$

where ξ is a nonincreasing differentiable function. He established a more general decay result for a wider class of relaxation functions. After that a series of papers using (1.2) has appeared, see for instance [14,20,21,29,31,33,35].

Guided by the experience with frictional damping initiated in the work of Lasiecka and Tataru [18], another step forward was done by considering relaxation functions satisfying

$$g'(t) \leq -\chi\left(g(t)\right). \tag{1.3}$$

This condition, where χ is a positive function, $\chi(0) = \chi'(0) = 0$, and χ is strictly increasing and strictly convex near the origin, with some additional constraints imposed on χ, was introduced by Alabau-Boussouira and Cannarsa [1] and was used then by several authors with different approaches. The result announced in [1], without proof, provides the decay rates expressed in terms of the relaxation kernel g in the case g satisfies the equality $g'(t) = -\chi\left(g(t)\right)$. While, for the case of inequality, [1] claims uniform decay of the energy without specifying the rate. For treatment of the general differential inequality (1.3), we refer to the work of Mustafa and Messaoudi [32] who provided explicit decay rates and also refer to [7,9,10,13,17,19,30,36] where decay results in terms of χ were obtained. Here, it should be mentioned that, in [19], it was the first time where Lasiecka and Wang established not only general but also optimal results in which the decay rates are characterized by an ODE of the same type as the one generated by the inequality (1.3) satisfied by g. Also, in [10], Cavalcanti et al. studied a quasilinear abstract viscoelastic system and followed the general philosophy of [19], but there were new technical details brought to the picture by the interaction between the nonlinearity and viscoelasticity which forced a new methodology and tricks.

Our aim in this work is to investigate (1.1) for relaxation functions g of more general type than the ones in (1.2) and (1.3). We consider the condition $g'(t) \leq -\xi(t)\chi\left(g(t)\right)$, where χ is increasing and convex **without any additional constraints**, overcome the difficulty brought by both the nonlinear source term and the dispersion term, and establish energy decay results that address both the optimality and generality. On the optimality level, the energy decay rates are consistent with the decay rates of g. We use minimal conditions and obtain explicit energy decay formulas which give the best decay rates expected under this level of generality. Our results substantially improve and generalize the earlier related results in the literature. The proof is based on the multiplier method and makes use of some properties of convex functions with some arguments from [16,18,32]. The paper is organized as follows. In section 2, we present our main result. Some technical lemmas are provided in section 3. The proof of the main result is given in section 4.

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