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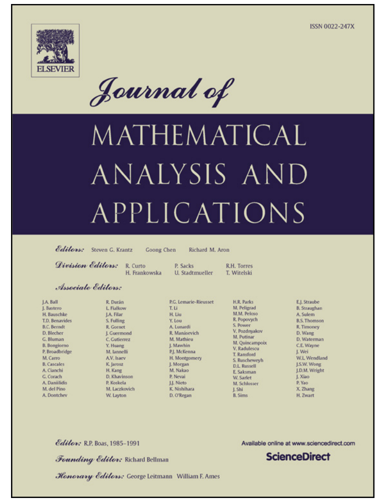

Approximation for non-smooth functionals of stochastic differential equations with irregular drift

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Approximation for non-smooth functionals of stochastic differential equations with irregular drift

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Abstract

This paper aims at developing a systematic study for the weak rate of convergence of the Euler-Maruyama scheme for stochastic differential equations with very irregular drift and constant diffusion coefficients. We apply our method to obtain the rates of approximation for the expectation of various non-smooth functionals of both stochastic differential equations and killed diffusion. We also apply our method to the study of the weak approximation of reflected stochastic differential equations whose drift is Hölder continuous.
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1 Introduction

Let $T > 0$ be fixed and $(X_t)_{0 \le t \le T}$ be the solution to

$$dX_t = b(X_t)dt + \sigma(X_t)dW_t, \quad X_0 = x_0 \in \mathbb{R}^d, \ 0 \le t \le T,$$

where W is a d-dimensional Brownian motion. The diffusion $(X_t)_{0 \le t \le T}$ is used to model many random dynamical phenomena in many fields of applications. In practice, one often encounters the problem of evaluating functionals of the type $\mathbb{E}[f(X)]$ for some given function $f : C[0,T] \to \mathbb{R}$. For example, in mathematical finance the function f is commonly referred as a *payoff* function. Since they are rarely analytically tractable, these expectations are usually approximated using numerical schemes. One of the most popular approximation methods is the Monte Carlo Euler-Maruyama method which consists of two steps:

1. The diffusion process $(X_t)_{0 \le t \le T}$ is approximated using the Euler-Maruyama scheme $(X_t^h)_{0 \le t \le T}$ with a small time step $h > 0$:

$$dX_t^h = b(X_{\eta_h(t)}^h)dt + \sigma(X_{\eta_h(t)}^h)dW_t, \quad X_0^h = x_0, \ \eta_h(t) = kh, \tag{1}$$

for $t \in [kh, (k+1)h)$, $k \in \mathbb{N}$.

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