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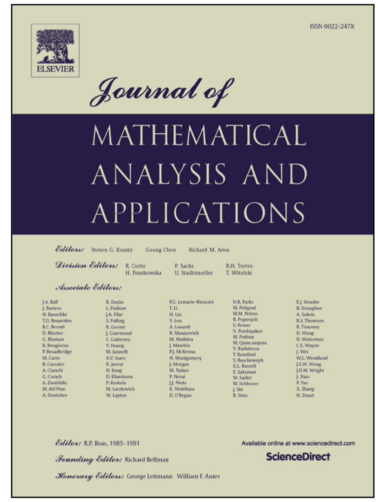

Continued fraction approximation for the Gamma function based on the Tri-gamma function

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Abstract

In this paper, we present a continued fraction product approximation for the Gamma function, via the Tri-gamma function. This approximation is fast in comparison with the recently discovered asymptotic series. We also establish the inequalities related to this approximation. Finally, some numerical computations are provided for demonstrating the superiority of our approximation.

1 Introduction

It is well known that we often need to deal with the problem of approximating the factorial function $n!$, and its extension to real numbers called the Gamma function, defined by

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt, \mathrm{Re}(x) > 0,$$

and the logarithmic derivatives of $\Gamma(x)$ are called the psi-gamma functions, denoted by

$$\psi(x) = \frac{d}{dx} \ln \Gamma(x) = \frac{\Gamma'(x)}{\Gamma(x)}.$$

For $x > 0$, the derivatives $\psi'(x)$ are called the Tri-gamma functions, while the derivatives $\psi^{(k)}(x), k = 1, 2, 3, ...$ are called the poly-gamma functions.

Mortici [1] proved that

$$\Gamma(x+1) = \sqrt{2\pi x} \left(\frac{x}{e}\right)^x \exp\left(\frac{1}{12}\psi'(x+1/2)\right) \exp h(x) \tag{1.1}$$

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