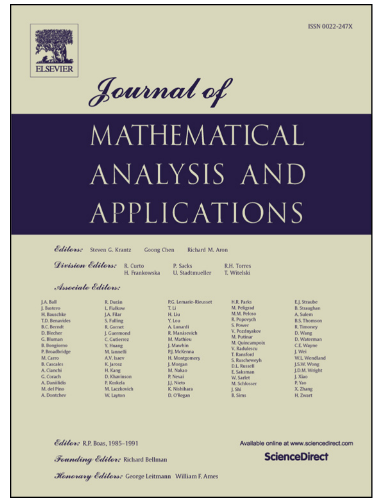

Sequences of dilations and translations in function spaces

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SEQUENCES OF DILATIONS AND TRANSLATIONS IN FUNCTION SPACES

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ABSTRACT. Let $f \in L^1[0,1]$ be a mean zero function and let f_n, $n = 1, 2, \dots$, be the dyadic dilations and translations of f. We investigate conditions on f, under which the linear operator T_f defined by $T_f h_n = f_n$, $n = 1, 2, \dots$, where h_n, $n = 1, 2, \dots$, are mean zero Haar functions, can be continuously extended to the closed linear span $[h_n]$ in a certain function space X. Among other results we prove that T_f is bounded in every symmetric space with nontrivial Boyd indices whenever $f \in BMO_d$ and f has "good" Haar spectral properties. In the special case of so-called Haar chaoses the above results can be essentially refined and sharpened. In particular, we find necessary and sufficient conditions, under which the operator T_f, generated by a Haar chaos f of order 1, is continuously invertible in L^p for all $1 < p < \infty$.

1. Introduction

Let a function $f \in L^1[0,1]$ have zero mean value, i.e.,

$$\int_0^1 f(t)\,dt = 0. \tag{1}$$

Definition 1. *The sequence of dilations and translations of f (or Haar affine system generated by f) consists of the functions*

$$f_n(t) := \begin{cases} f(2^k t - j), & \text{if } t \in [\frac{j}{2^k}, \frac{j+1}{2^k}], \\ 0, & \text{otherwise}, \end{cases}$$

where $n = 2^k + j$, $k = 0, 1, \dots$ and $j = 0, \dots, 2^k - 1$.

In particular, if f is the Haar function

$$h(t) = \begin{cases} 1, & 0 < t < \frac{1}{2}, \\ -1, & \frac{1}{2} < t < 1, \\ 0, & \text{otherwise}, \end{cases}$$

we get the classical Haar system $\{h_n\}_{n=1}^\infty$ (without the function $h_0(t) = 1$), normalized in L^∞ (cf. [14, Chapter 3], [17, 2c], [19, Chapter 1], [21]).

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Key words and phrases. sequence of dilations and translations, Haar functions, symmetric space, BMO space, H^1 space, interpolation of operators.

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