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Asymptotic expansions for Laplace transforms of Markov processes

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ABSTRACT

Let μ^ϵ be the probability measures on $D[0,T]$ of suitable Markov processes $\{\xi_t^\epsilon\}_{0\leq t\leq T}$ (possibly with small jumps) depending on a small parameter $\epsilon > 0$, where $D[0,T]$ denotes the space of all functions on $[0,T]$ which are right continuous with left limits. In this paper we investigate asymptotic expansions for the Laplace transforms $\int_{D[0,T]} \exp\{\epsilon^{-1}F(x)\}\mu^\epsilon(dx)$ as $\epsilon \to 0$ for smooth functionals F on $D[0,T]$. This study not only recovers several well-known results, but more importantly provides new expansions for jump Markov processes. Besides several standard tools such as exponential change of measures and Taylor's expansions, the novelty of the proof is to implement the expectation asymptotic expansions on normal deviations which were recently derived in [13].

1. Introduction

Suppose that μ^ϵ is a family of probability measures on a metric space $\mathbb{X}$ depending on a small parameter ϵ, the asymptotic behaviors of $\int_{\mathbb{X}} \exp\{\epsilon^{-1}F(x)\}\mu^\epsilon(dx)$ as $\epsilon \to 0$ have been attracting much attention for suitable functions F on $\mathbb{X}$, due to their importance in many areas of mathematics and physics. In particular they have a direct connection with large deviation theory through the Varadhan's integral lemma (cf. [10] and [3, Section 4.3]). More precisely, a large deviation principle for the family of measures μ^ϵ corresponds to the rough asymptotics, namely $\ln \int_{\mathbb{X}} \exp\{\epsilon^{-1}F(x)\}\mu^\epsilon(dx)$. The aim of this paper is to investigate asymptotic expansions of such function space integrals in the following form: for suitable family of measures μ^ϵ and smooth functions $F(x)$,

$$\int_{\mathbb{X}} \exp\{\epsilon^{-1}F(x)\}\mu^\epsilon(dx) = \exp\left\{K\cdot\epsilon^{-1}\right\}\left[\sum_{0\leq i\leq n} K_i\cdot\epsilon^{i/2} + o(\epsilon^{n/2})\right] \tag{1.1}$$

where K, K_i and n are appropriate constants.

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It is clear that the family of probability measures μ^ϵ plays an essential role and highly affects the validity of the asymptotic expansions (1.1). Indeed, for every fixed $\epsilon > 0$ if μ^ϵ is the probability distribution of the process $\{\sqrt{\epsilon} \cdot W_t\}_{t\in[0,T]}$ on the space $\mathbb{X} = C[0,T]$, where $\{W_t\}$ is a standard Wiener process and $C[0,T]$ is the continuous function space on $[0,T]$, then it was proved in [8] that (1.1) holds with an integer n depending on the smoothness of F. More generally if μ^ϵ takes the form $\mu^\epsilon = \sqrt{\epsilon} \cdot \mu$ with μ denoting a suitable Gaussian measure on a Banach space $\mathbb{X}$, then in [4] the expansions (1.1) were achieved. The proofs of these aforementioned results are based on two standard and important tools: exponential change of measures (which is commonly used in large deviation theory) and Taylor's expansions of functionals. Therefore, it is inevitable in the proofs that functional derivatives (in the sense of Fréchet differentiability) and their estimates appear, such as $F^{(j)}(x_0)(x^{\otimes^j})$ which is the j-th Fréchet derivative of F at the point $x_0 \in \mathbb{X}$ in the directions $x \in \mathbb{X}$.

To go beyond the family of measures with the special form $\mu^\epsilon = \sqrt{\epsilon} \cdot \mu$, one needs to overcome at least one difficulty: estimation in terms of ϵ on the functional derivatives $F^{(j)}(\phi_0)(\xi^{\epsilon\otimes^j})$ with ξ^ϵ being the stochastic processes associated with the measures μ^ϵ. We note that it is trivial to have estimates in terms of ϵ on the functional derivatives if we consider the special form $\mu^\epsilon = \sqrt{\epsilon} \cdot \mu$ (or equivalently $\xi^\epsilon = \sqrt{\epsilon} \cdot \xi$) for μ (or equivalently for ξ) being a Gaussian measure on a Banach space, since in this case the derivatives simply become

$$F^{(j)}(\phi_0)((\sqrt{\epsilon} \cdot \xi)^{\otimes^j}) = \epsilon^{j/2}\, F^{(j)}(\phi_0)(\xi^{\otimes^j}).$$

Along this direction, several extensions of the asymptotic expansions (1.1) to other families of measures are obtained (such as families of diffusion processes, cf. [1] and references therein, and families of Itô functionals of Brownian rough paths, cf. [5]) by using *stochastic Taylor's expansion* to overcome such a difficulty. However extensions of such a stochastic Taylor's expansion in order to include more general families of measures μ^ϵ (especially the ones which will be used throughout the paper) are not easy and convenient (see for instance [5, Section 5] for the long and elaborated estimates on the remainder terms).

In this paper, the family of probability measures μ^ϵ under consideration are those on the space $D[0,T]$ corresponding to a family of locally infinitely divisible Markov processes $\{\xi_t^\epsilon\}_{0\le t\le T}$ whose precise definition is given in Section 2.1. Such a family contains many well-known stochastic processes such as Wiener process and Lévy process, making it natural to be investigated. As explained above, estimation in terms of ϵ on the functional derivatives $F^{(j)}(\phi_0)(\xi^{\epsilon\otimes^j})$ is essential to achieve the asymptotic expansions (1.1) for this family μ^ϵ. In this paper, instead of trying to modify and employ the stochastic Taylor's expansion, we will for the first time employ a recent result in [13] regarding the expectation asymptotic expansions on normal deviations to overcome such a difficulty. Besides this difficulty, there are many non-light technical estimates to be established involving moments and upper tail probabilities of the Markov processes ξ^ϵ (see Sections 4.3–4.6), and we achieve them by intensively using large deviation techniques for Markov processes developed in [11].

The rest of this paper is organized as follows. In Section 2 the precise definition of a locally infinitely divisible Markov process is given together with an associated standard tool which will be used frequently throughout the paper: exponential change of measures (or Cramér's transformation). The main result of the paper is then formulated in Section 3, and several examples are included there as well. Section 4 contains a detailed proof of the main result, while some less relevant technical arguments are postponed to Section 5.

2. Locally infinitely divisible Markov processes

2.1. Definition

In this section we use *compensating operator* and *generating operator* to characterize a Markov process. If $(\xi_t, \mathbb{P}_{s,x}), t \in [s,T]$, is a real-valued Markov process (the subscript $_{s,x}$ means the process starts from x at