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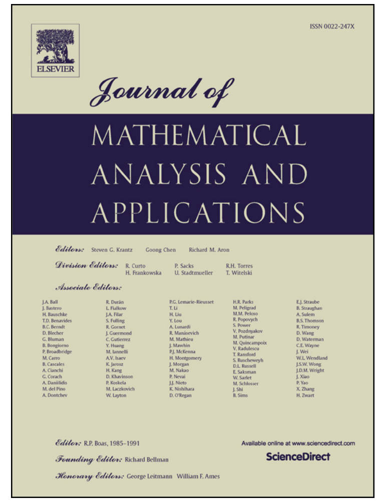

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Existence and blow-up rate of large solutions of $p(x)$-Laplacian equations with gradient terms

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Abstract

In this paper we investigate boundary blow-up solutions of the problem

$$\begin{cases} -\Delta_{p(x)}u + f(x,u) = \pm K(x)|\nabla u|^{m(x)} \text{ in } \Omega, \\ u(x) \to +\infty \quad \text{as } d(x,\partial\Omega) \to 0, \end{cases}$$

where $\Delta_{p(x)}u = \operatorname{div}(|\nabla u|^{p(x)-2}\nabla u)$ is called the $p(x)$-Laplacian. Our results extend the previous work [25] of Y. Liang, Q.H. Zhang and C.S. Zhao from the radial case to the non-radial setting, and [43] due to Q.H. Zhang and D. Motreanu from the assumption that $K(x)|\nabla u(x)|^{m(x)}$ is a small perturbation, to the case in which $\pm K(x)|\nabla u|^{m(x)}$ is a large perturbation. We provide an exact estimate of the pointwise different behavior of the solutions near the boundary in terms of $d(x,\partial\Omega)$ and in terms of the growth of the exponents. Furthermore, the comparison principle is no longer applicable in our context, since $f(x,\cdot)$ is not assumed to be monotone in this paper.

Key Words: $p(x)$-Laplacian; subsolution; supersolution; boundary blow-up solution; singularity.

Mathematics Subject Classification(2010): 35J25; 35J62

1 Introduction

Let $\Omega \subset \mathbb{R}^N$, $N \geq 2$, be a bounded domain with C^2 boundary $\partial\Omega$. We consider boundary blow-up solutions of the variable exponent elliptic problem

$$\begin{cases} -\Delta_{p(x)}u + f(x,u) = \pm K(x)|\nabla u|^{m(x)} \text{ in } \Omega, \\ u(x) \to +\infty \quad \text{as } d(x,\partial\Omega) \to 0, \end{cases} \tag{P$_\pm$}$$

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