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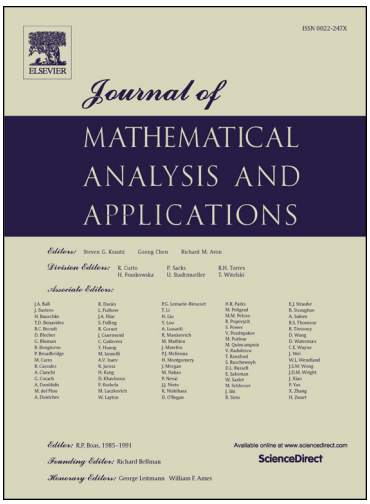

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A note on Markov perfect equilibria in a class of non-stationary stochastic bequest games

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Abstract. In this note, we prove the existence of a Markov perfect equilibrium in a non-stationary version of a paternalistic bequest game. The method we advocate is general and allows to study models with unbounded state space and unbounded utility functions. We cover both, the stochastic and deterministic cases. We provide a characterization of the set of all Markov perfect equilibria by means of a set-valued recursive equation involving the best response operator. In the stationary case, we show that there exists a set of strategies that is invariant under the best response mapping.

Keywords: Stochastic game; Non-stationary bequest game; Markov perfect equilibrium

1 Introduction

Since the seminal papers of Phelps and Pollak [16], Kohlberg [11], Bernheim and Ray [7], or Leininger [12] researchers study a class of paternalistic bequest economies. Specifically, the economy they consider consists of a sequence of generations, each living one period, and deriving utility from its own consumption, as well as that of a successor generation. At the beginning of each period t, generation receives an endowment of a single homogeneous good, which for $t \geq 2$ is the output from a bequest left by the previous generation. This endowment is divided between consumption and investment.

From a game-theoretic point of view this economy is represented by an infinite horizon "bequest game" on an uncountable, possibly unbounded state space with countably many short-lived players (generations). A natural solution concept for this class of games is the subgame (or Markov) perfect equilibrium. First results on the existence of Markov perfect equilibria in bequest games with deterministic transitions were proved (by different methods) in [7] and [12]. Although the model of the bequest game looks simple, the proofs given in the aforementioned papers are quite involved and based on some technical tricks. A simpler and more transparent proof is given in [4].

Extensions of the simple bequest game model involving more than one descendant for each generation and stochastic transitions were considered by many authors. A survey