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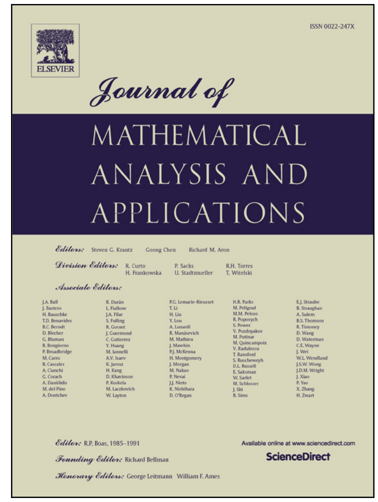

Chaotic semigroups from second order partial differential equations

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CHAOTIC SEMIGROUPS FROM SECOND ORDER PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. We give general conditions on given parameters to ensure Devaney and distributional chaos for the solution C_0-semigroup corresponding to a class of second-order partial differential equations. We also provide a critical parameter that led us to distinguish between stability and chaos for these semigroups. In the case of chaos, we prove that the C_0-semigroup admits a strongly mixing measure with full support. We also give concrete examples of partial differential equations, such as the telegraph equation, whose solutions satisfy these properties.

1. Introduction

The phenomenon of chaos is usually identified with nonlinear phenomena, but chaos also appears in linear dynamical systems provided that the underlying space is infinite-dimensional. The theory of chaos in finite-dimensional dynamical systems has been well-developed and includes both discrete maps and systems of ordinary differential equations. This theory has led to important applications in physics, chemistry, biology, and engineering. However, for a long period of time, there was no theory of chaos for partial differential equations (PDE's). In terms of applications, most of the important natural phenomena are described by PDE's: nonlinear wave equations, Maxwell equations, Navier-Stokes equations, and so on. These equations model a wide variety of phenomena in cell proliferation, electrostatics, electrodynamics, elasticity, fluid flow, heat conduction, sound propagation, or traffic modelling.

The study of C_0-semigroups has been widely identified with partial of parabolic and hyperbolic type differential equations. It is now well known that the solutions of these equations can be represented in terms of C_0-semigroups [27]. They permit the solution to the corresponding abstract Cauchy problem to be described in a broader setting, for instance, including non-differentiable integrable functions as initial conditions.

In this paper, we provide a new insight into the chaotic behavior of any C_0-semigroup that is solution of a certain class of second order partial differential equations, considering both Devaney and distributional chaos. The study will be carried out on Herzog type spaces [32]. Herzog's result was later improved in [24].

These spaces consist of analytic functions regulated by a parameter, or a tuner, that allows their growth at infinity to be controlled. They were initially introduced in order to study the universality of the solution operators of the heat equation. In [16], Chan & Shapiro studied the dynamics of the translation operator on spaces of analytic functions of slow growth and characterized when the derivative operator was bounded on these spaces. Then, since the derivative operator is the infinitesimal generator of the translation semigroup, we can conclude that the translation semigroup is uniformly continuous and all its operators can be obtained via the exponential formula. See for instance [27, Th. 3.7]. Interesting constructions and counterexamples have been given in the framework of certain subspaces of analytic functions, see for instance [10, 40, 14]. Godefroy & Shapiro also considered Hardy and Bergman spaces for studying the dynamics of shift operators, see [28].

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