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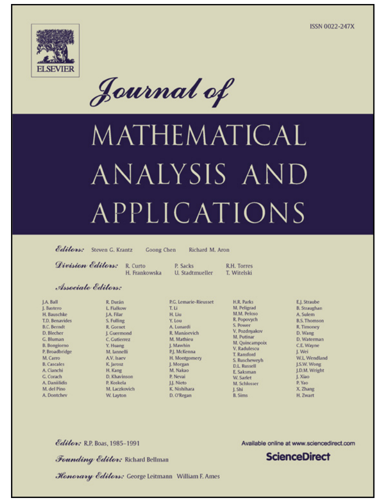

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INVARIANT MEASURES ON MULTI-VALUED FUNCTIONS

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ABSTRACT. In this paper we consider the question of under which conditions multi-valued dynamical systems admit invariant measures. We give results on the existence of invariant measures with full support on orbit spaces of multi-valued dynamical systems. We use these measures on the orbit space to induce measures on the original dynamical system. We focus on the question of when a non-atomic invariant measure on the orbit space induces an atomic invariant measure on the multi-valued dynamical system. This phenomenon is an indicator of complicated multi-periodic behavior.

1. INTRODUCTION

In the context of discrete dynamical systems of compact metric spaces, for instance $f : X \to X$, with f continuous, there are many results regarding the existence of measures on X that are invariant with respect to f. In particular, Denker, Grillenberger, and Sigmund show that if f has the specification property, then the space of invariant measures on (X, f) has a dense G_δ set of non-atomic measures with full support that are invariant with respect to f, [3]. In the context of multi-valued functions, relatively little is known about the existence of invariant measures. Motivated by models from mathematical economics, Kennedy, Raines, and Stockman ask under what conditions an upper-semi-continuous (usc) multi-valued function admits an invariant measure, [5], and they give specific attention to non-atomic invariant measures with full support. Kennedy, Raines, and Stockman were interested specifically in the inverse limit space generated by a multi-valued dynamical system as it is the natural space of solutions for certain models from mathematical economics, [6], [10]. It is also not uncommon for models from mathematical economics to display Euler equation branching in a continuous-time setting, [7], [11]. The discrete time equivalent to Euler equation branching is a multi-valued dynamical systems (MVDS).

In this paper we examine the construction of invariant measures for MVDS's. In the Kennedy, Raines and Stockman paper, mentioned above, the authors assumed that the dynamical system (X, f) had an invariant measure μ and then used that measure to induce a measure $\hat{\mu}$ on the associated inverse limit space. They then examined under what conditions the induced measure is invariant, non-atomic and has full support. In this paper we reverse this process. Assuming the MVDS, (X, F), has the specification property, then so will its associated inverse limit space, $\varprojlim F$, and forward orbit space, $\mathrm{Orb}^+(F)$ (since the inverse of a MVDS is a well-defined MVDS we focus on $\mathrm{Orb}^+(F)$ in this paper instead of $\varprojlim F$). Since the

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