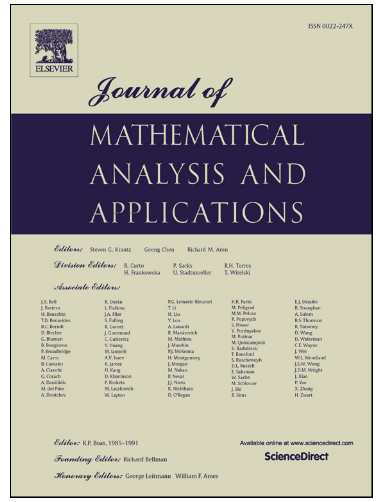

Dependence of eigenvalues of fourth-order differential equations with discontinuous boundary conditions on the problem

Xiao-xia Lv, Ji-jun Ao, Anton Zettl

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Dependence of eigenvalues of fourth-order differential equations with discontinuous boundary conditions on the problem

Xiao-xia Lv, Ji-jun Ao, and Anton Zettl

Abstract. Fourth-order boundary value problems with discontinuous boundary conditions are studied. We prove that the eigenvalues depend not only continuously but smoothly on the coefficients and on the boundary conditions and find formulas for the derivatives with respect to each of these parameters.

1. Introduction

For classical regular second order self-adjoint Sturm-Liouville problems the dependence of the eigenvalues on the problem is now, due to some surprisingly recent results given the long history of these problems, well understood: see [**18**] and the survey paper [**19**]. Such a problem consists of
the differential equation

$$My = -(py')' + qy = \lambda wy,\ \lambda \in \mathbb{C},\ \text{on} \quad J = (a, b),\ -\infty < a < b < \infty, \tag{1.1}$$

with coefficients satisfying:

$$r = 1/p,\ q,\ w \in L^1(J, \mathbb{R}),\ p > 0,\ w > 0\ a.e.\ on\ J, \tag{1.2}$$

and two-point boundary conditions

$$AY(a) + BY(b) = 0,\ Y = \begin{bmatrix} y \\ (py') \end{bmatrix}, \tag{1.3}$$

satisfying

$$A,\, B \in M_2(\mathbb{C}),\ \ A\,E\,A^* = B\,E\,B^*,\ rank\,(A:B) = 2,\ E = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{1.4}$$

Here $M_2(\mathbb{C})$ denotes the 2×2 matrices over the complex numbers $\mathbb{C}$ and $L^1(J, \mathbb{R})$ denotes the real valued Lebesgue integrable functions on the (entire) interval J. We also use the notation $M_2(\mathbb{R})$ for the real 2×2 matrices. Throughout, $\mathbb{N}_0 := \{0, 1, 2, \ldots\}$, $\mathbb{R}$ and $\mathbb{C}$ denote the real and complex numbers, respectively.

The boundary conditions (1.3), (1.4) are homogeneous and thus clearly invariant under left multiplication by a nonsingular matrix. It is well known that the

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