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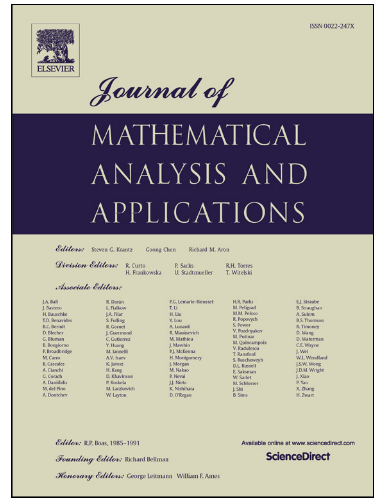

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Well-posedness and regularity of the generalized Burgers equation in periodic Gevrey spaces

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Abstract

We consider the generalized Burgers equation in a class of Gevrey functions $G^{\sigma,\delta,s}(\mathbb{T})$. We show that the generalized Burgers equation is well–posed in this space. Furthermore, we show that the solution is Gevrey-σ in the spacial variable and Gevrey-2σ in the time variable.

Keywords: Burgers equation; Gevrey regularity; Initial value problem; Multilinear estimates; Sobolev spaces; Uniform radius of analyticity
2010 MSC: Primary: 35Q53

1. Introduction

For $k = 1, 2, 3, \ldots$ we consider the initial-value problem for the generalized Burgers equation (k-gBurgers)

$$\begin{cases} \partial_t u = \partial_x^2 u + u^k \partial_x u, \\ u(x,0) = u_0(x), \end{cases} \quad x \in \mathbb{T}, \ t \in \mathbb{R} \tag{1.1}$$

and study its well–posedness and the regularity properties of the solution when the initial data belong to a class of analytic Gevrey spaces. These spaces are defined by the norm

$$\|f\|^2_{\dot{G}^{\sigma,\delta,s}(\mathbb{T})} = \sum_{n\neq 0} |n|^{2s} e^{2\delta|n|^{1/\sigma}} |\widehat{f}(n)|^2 < \infty \tag{1.2}$$

for any $s \in \mathbb{R}$, $\delta > 0$ and $\sigma \geq 1$.

A function in $\dot{G}^{1,\delta,s}(\mathbb{T})$ is a real analytic function which has a holomorphic extension on a symmetric strip of width 2δ in the complex plane. For an analytic periodic function φ, there exists $\delta_0 > 0$ such that $\varphi \in \dot{G}^{1,\delta_0,s}(\mathbb{T})$. Thus, well-posedness in these spaces shows that the width of the strip of analyticity does not collapse during the time interval of existence. It is clear that the dominant weighting of these L^2 spaces is by the exponential term, i.e. $e^{2\delta|n|^{1/\sigma}}$. We include the Sobolev weight, $|n|^{2s}$, in the definition of these spaces simply to simplify the multilinear estimates we will need. Therefore, in our proofs, we will fix the index $s = s_k = \frac{1}{2} - \frac{1}{k}$ to simplify our computations.

Using these spaces, our main result about the well-posedness of k-gBurgers in analytic Gevrey spaces reads as follows.

Theorem 1. *Let $\sigma \geq 1$, $\delta > 0$ and $s_k = \frac{1}{2} - \frac{1}{k}$. Given small initial data, $u_0(x)$, in the Gevrey spaces $\dot{G}^{\sigma,\delta,s_k}(\mathbb{T})$, for any $T > 0$ the initial-value problem for the k-gBurgers equation* (1.1) *is well-posed in the space $C([0,T]; \dot{G}^{\sigma,\delta,s_k})$.*