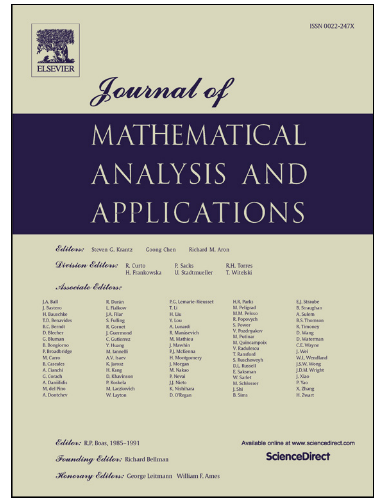
Accepted Manuscript



Asymptotic behaviors of the full quantum hydrodynamic equations

Xueke Pu, Xiuli Xu

PII: S0022-247X(17)30416-X
DOI: http://dx.doi.org/10.1016/j.jmaa.2017.04.053
Reference: YJMAA 21340

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 6 January 2017

ASYMPTOTIC BEHAVIORS OF THE FULL QUANTUM HYDRODYNAMIC EQUATIONS

XUEKE PU AND XIULI XU

ABSTRACT. The full hydrodynamic equations with quantum effects are studied in this paper. We obtain global solutions and optimal convergence rates by pure energy method provided the initial perturbation around a constant state is small enough. In particular, the optimal decay rates of the higher-order spatial derivatives of solutions are obtained.

1. INTRODUCTION

In this paper, we consider the following full quantum hydrodynamic system [10]

$$\begin{cases} n_t + \mathrm{div}(nu) = 0, & \text{(1.1a)} \\ u_t + u\cdot\nabla u + \dfrac{1}{mn}\nabla(nT) - \dfrac{\hbar^2}{12m^2 n}\mathrm{div}\big(n(\nabla\otimes\nabla)\log n\big) = \dfrac{1}{mn}\mathrm{div}\mathbb{S}, & \text{(1.1b)} \\ T_t + u\cdot\nabla T + \dfrac{2}{3}T\mathrm{div}u - \dfrac{2}{3n}\mathrm{div}(\kappa\nabla T) - \dfrac{\hbar^2}{18mn}\mathrm{div}(n\nabla\mathrm{div}u) & \\ \qquad\qquad = \dfrac{2}{3mn}\mathrm{div}(u\mathbb{S} - u\cdot\mathrm{div}\mathbb{S}), & \text{(1.1c)} \end{cases}$$

where the unknowns (n, u, T) are functions of $(x, t) \in \mathbb{R}^3 \times \mathbb{R}^+$, representing the density, velocity and temperature, respectively. Here, the viscous stress tensor $\mathbb{S}$ is defined by

$$\mathbb{S} = \mu(\nabla u + (\nabla u)^{\mathrm{T}}) + \lambda(\mathrm{div}u)\mathbb{I},$$

where $\mathbb{I}$ is the $d\times d$ identity matrix and $\mu > 0$ and λ are the first and the second coefficients of viscosity, satisfying the usual condition $2\mu + 3\lambda > 0$. In the above equation, m is the effective electron mass, $\hbar > 0$ is the Plank constant and κ is the thermal conductivity. The quantum effects terms (those with coefficient $\hbar^2$) are included in not only the momentum equation but also the energy equation, which makes the system more interesting as well difficult to analyze in mathematics.

Without the quantum effects, the above system (1.1) is well known as the full Navier-Stokes equation and was extensively studied since the beginning of the twentieth century. The interested reader may refer to the monograph of Lions [15] and the references therein. So we only put emphasis on the two quantum terms in (1.1). The quantum terms date back to Wigner [26], where quantum corrections were considered for the thermodynamic equilibrium. These quantum correction terms are closely related to Bohm potential [3]. The quantum correction to the stress tensor was proposed by Ancona and Tiersten [2] and Ancona and Iafrate [1] on the Wigner formalism. By a moment expansion of the Wigner-Boltzmann equation and an expansion of the thermal equilibrium Wigner distribution function to $O(\hbar^2)$,

2000 *Mathematics Subject Classification.* 35Q53; 35Q35.

Key words and phrases. quantum hydrodynamic equations; energy method; negative Sobolev space; optimal decay rates; global existence.

This work is supported in part by NSFC (11471057) and the Fundamental Research Funds for the Central Universities (106112016CDJZR105501).