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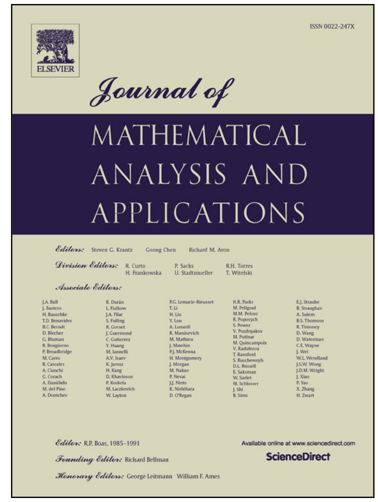

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Yiannis Karagiorgos, Nikos Yannakakis

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A Neumann problem for the $p(x)$-Laplacian with $p = 1$ in a subdomain

Yiannis Karagiorgos

Department of Mathematics, National Technical University of Athens, Zografou campus, Athens 15780, Greece

Nikos Yannakakis*

Department of Mathematics, National Technical University of Athens, Zografou campus, Athens 15780, Greece

Abstract

In this paper we study a Neumann problem with non-homogeneous boundary conditions for the $p(x)$-Laplacian. In particular we assume that $p(\cdot)$ is a step function defined in a domain Ω and equals to 1 in a subdomain Ω_1 and 2 in its complementary Ω_2. By considering a suitable sequence p_k of variable exponents such that $p_k \to p$ and replacing p with p_k in the original problem, we prove the existence of a solution u_k for each of those intermediate ones. We also show, that under a hypotheses concerning the boundary data g, the limit of the sequence (u_k) is a function u, which belongs to the space of functions of bounded variation and is a solution to the original $p(\cdot)$-problem.

Keywords: Neumann problem, $p(x)$-Laplacian, 1-Laplacian, BV space.
2000 MSC: 35J20, 35J60

1. Introduction

Consider the following Neumann problem

$$\begin{cases} -\Delta_{p(x)} u(x) = 0, & x \in \Omega \\ |\nabla u(x)|^{p(x)-2} \frac{\partial u}{\partial \nu}(x) = g(x), & x \in \partial\Omega, \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain and $N \geq 2$.

$$\Delta_{p(x)} u := \operatorname{div}(|\nabla u|^{p(x)-2} \nabla u)$$

is the $p(x)$-Laplacian operator which is the variable exponent version of the p-Laplacian. Also, $g \in L^\infty(\partial\Omega)$ and satisfies $\int_{\partial\Omega} g \, d\mathcal{H}^{N-1} = 0$. Note that this latter condition is necessary, since otherwise problem (P) has no solution.

*Corresponding author
Email addresses: ykarag@math.ntua.gr (Yiannis Karagiorgos), nyian@math.ntua.gr (Nikos Yannakakis)