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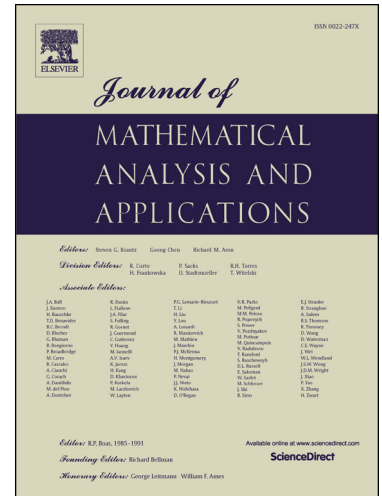
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Bounds on ratios of modified Bessel functions with complex arguments

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Abstract

A simple uniform bound for the ratio of two modified spherical Bessel functions is derived. The arguments of the two functions are complex but their ratio is real.

Keywords: spherical Bessel function, Bessel polynomial, generalized hypergeometric function, special function

1. Introduction

In a recent investigation in the context of acoustic scattering, we encountered a Volterra integral equation of the second kind,

$$u(x) - \int_x^\infty \mathcal{K}(x, y) u(y) dy = f(x), \quad x > 1, \quad (1)$$

where $f(x)$ is given and $u(x)$ is to be found. The kernel is given by

$$\mathcal{K}(x, y) = \int_x^y \left(\frac{W(y)}{W(\eta)} \right)^2 d\eta. \quad (2)$$

where

$$W(\xi) = \xi^{1/2} K_\nu(\mu\xi), \quad (3)$$

K_ν is a modified Bessel function, and the parameters μ and ν will be specified shortly. Usually, Volterra integral equations of the second kind (such as (1)) can be solved by iteration. To justify this approach, a bound on $|\mathcal{K}(x, y)|$ is needed.

Fortunately, there is a recent paper by Baricz [1] containing a thorough review of the literature on known bounds on ratios of modified Bessel functions. From [1, eqn (3.6)],

$$\frac{K_\nu(x)}{K_\nu(y)} > e^{y-x} \left(\frac{y}{x} \right)^{1/2}, \quad |\nu| > \frac{1}{2}, \quad 0 < x < y.$$

Hence

$$F(y, \eta) \equiv \frac{W(y)}{W(\eta)} = \frac{y^{1/2} K_\nu(\mu y)}{\eta^{1/2} K_\nu(\mu \eta)} < e^{\mu(\eta-y)}, \quad 0 < \eta < y. \quad (4)$$

This holds for real μ with $\mu > 0$. Using the bound (4) in (2),

$$0 < \mathcal{K}(x, y) < \int_x^y e^{2\mu(\eta-y)} d\eta = \frac{1}{2\mu} \left(1 - e^{-2\mu(y-x)} \right) \leq \frac{1}{2\mu},$$

as $y \geq x$, and this uniform bound can be used to justify an iterative scheme.

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