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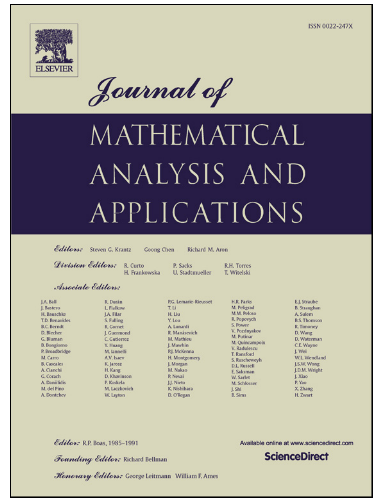

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GREEDY ALGORITHM WITH REGARD TO THE NEEDLET SYSTEM ON THE SPHERE

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Abstract. Order estimates of the best m-term approximation for the Besov classes and the Sobolev classes with regard to the dictionary Ψ on the sphere are established, where Ψ consists of needlets which are highly localized radial polynomials on the sphere with centers at the nodes of a suitable cubature rule. Moreover, it is shown that for these classes the orders of the best m-term approximation can be achieved by simple greedy type algorithm.

1. Introduction

Let $\mathbb{S}^d := \left\{x = (x_1, x_2, \dots, x_{d+1}) \in \mathbb{R}^{d+1} : \sum_{j=1}^{d+1} x_j^2 = 1\right\}$ be the unit sphere in $\mathbb{R}^{d+1}$ endowed with the usual rotation invariant Lebesgue measure $d\sigma(x)$ normalized by $\int_{\mathbb{S}^d} d\sigma(x) = 1$. Given $1 \le p < \infty$, we denote by $L_p \equiv L_p(\mathbb{S}^d)$ the usual Lebesgue space on $\mathbb{S}^d$ endowed with the norm

$$\|f\|_p := \left(\int_{\mathbb{S}^d} |f(x)|^p \, d\sigma(x)\right)^{1/p}, \quad 1 \le p < \infty.$$

For $p = \infty$ we assume that $L_\infty(\mathbb{S}^d)$ is replaced by the space $C(\mathbb{S}^d)$ of continuous functions on $\mathbb{S}^d$ equipped with the uniform norm. Given $\alpha > 0$ and $1 \le p, \theta \le \infty$, we denote by $B^\alpha_{p\,\theta} \equiv B^\alpha_{p\,\theta}(\mathbb{S}^d)$ the Besov space on the sphere, and by $W^\alpha_p \equiv W^\alpha_p(\mathbb{S}^d)$ the Sobolev space on the sphere (see the definition of $B^\alpha_{p\,\theta}$ and W^α_p in Section 2), and by $BB^\alpha_{p\theta}$ and BW^α_p the unit balls of $B^\alpha_{p\theta}$ and W^α_p.

Let D be a dictionary in a Banach space $(X, \|\cdot\|_X)$, i.e., $\operatorname{span} D$ constitutes a dense subset of X. We define the best m-term approximation of a given function $f \in X$ and a given function class $F \subset X$ with regard to the dictionary D by

$$\sigma_m(f, D)_X := \inf_{g_j \in D, c_j \in \mathbb{R},\, j=1,\dots,m} \left\| f - \sum_{j=1}^m c_j g_j \right\|_X,$$

and

$$\sigma_m(F, D)_X := \sup_{f \in F} \sigma_m(f, D)_X,$$

respectively. Nonlinear m-term approximation is important in applications in image and signal processing (see [5, 13]).

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