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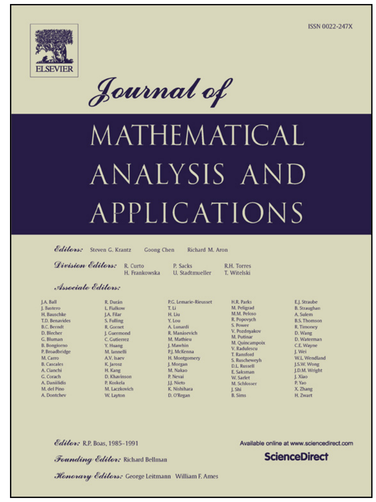

Sign-changing solutions for resonant Neumann problems

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Sign-changing solutions for resonant Neumann problems[☆]

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Abstract

We consider semilinear Neumann problems at resonance. We are concerned with two distinct cases. In the first one, the potential function is unbounded and indefinite. In the second case, the potential function is bounded and the reaction exhibits a concave parametric team near the origin. We prove two existence theorems producing sign-changing solutions. The first main result is a complement of the results obtained by Papageorgiou and Rădulescu [Trans. Amer. Math. Soc. 367 (2015) 8723-8756]. The second theorem gives an answer to the open question raised by Candito, D'Aguí and Papageorgiou [Topol. Methods Nonlinear Anal. 47 (2016) 289-317]. Our approach uses variational methods together with flow invariance arguments.

Keywords: Sign-changing solutions; Indefinite and unbounded potential; Resonance; Gradient flow; Concave nonlinearity.
Mathematics Subject Classification: 35J20; 35J60; 58E05

1. Introduction

Let $\Omega \subseteq \mathbb{R}^N$ be a bounded domain with a C^2-boundary $\partial\Omega$. In this paper we study the following semilinear Neumann problem:

$$\begin{cases} -\Delta u(z) + \beta(z)u(z) = \lambda \left|u(z)\right|^{q-2} u(z) + f(z, u(z)) & \text{in } \Omega, \\ \frac{\partial u}{\partial n} = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

for $\beta \in L^s(\Omega)$ with $s > N$, and $n(\cdot)$ the outward unit normal on $\partial\Omega$. Moreover, $\lambda \geq 0$ is a parameter and $q \in (1, 2)$. So, the term $\lambda \left|x\right|^{q-2} x$ with $\lambda > 0$ is strictly sublinear (concave term). In addition, we assume that the term $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function (i.e., for all $x \in \mathbb{R}$, the function $z \to f(z, x)$ is measurable and for almost all $z \in \Omega$, the function $x \to f(z, x)$ is continuous) which exhibits linear growth near $\pm\infty$. We allow for resonance to occur with respect to any nonnegative nonprinciplal eigenvalue of $(-\Delta + \beta(\cdot), H^1(\Omega))$.

The aim of this work is to prove two existence theorems producing sign-changing solutions for problem (1). In the first theorem, we assume that $\lambda = 0$, and $\beta(\cdot)$ is indefinite and unbounded.

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