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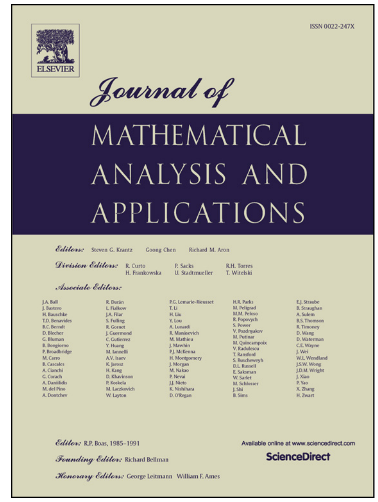

Boundary bubbling solutions for a supercritical Neumann problem with mixed nonlinearities

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BOUNDARY BUBBLING SOLUTIONS FOR A SUPERCRITICAL NEUMANN PROBLEM WITH MIXED NONLINEARITIES

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Abstract. In this paper, we investigate the existence of boundary bubbling solutions for a supercritical Neumann problem with mixed nonlinearities under a suitable assumption on the mean curvature of the boundary $\partial\Omega$, the behavior of the solutions is as a tower of k bubbles when the exponent goes to Sobolev critical exponent, and the blow-up point is a critical point of the mean curvature.

MR(2010) Subject Classification: 35B33, 35J25 , 35J61.

Keywords: Boundary bubbling solutions; Neumann boundary problem; Lyapunov-Schmidt reduction.

1. Introduction

In this paper, we are concerned with the following Neumann boundary problem

$$\begin{cases} -\Delta u + u = u^p + u^q & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$, $N \geq 4$, $1 < q < p$, and ν is the outward normal on $\partial\Omega$.

If $p = q$, problem (1.1) reduces to the following problem

$$\begin{cases} -\Delta u + u = u^p & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.2}$$

where Ω is a smooth bounded domain. If $1 < p < \frac{N+2}{N-2}$, (1.2) is subcritical problem with the Neumann boundary condition. It was first studied by Lin, Ni and Takagi [21]. Many other existence results were obtained, see [17], [18], [22], [23] and references therein.

If $p = \frac{N+2}{N-2}$, (1.2) becomes the critical Neumann boundary problem, the lack of compactness of Sobolev embedding makes it harder to apply variational arguments. In order to find nontrivial solutions, one has to deal with the lack of compactness. In [33], Wang obtained the existence of non-constant least energy solution of (1.2) for $p = \frac{N+2}{N-2}$. Many results have been obtained for the critical case, we refer to [7], [20], [29], [30] and references therein.

If p is supercritical in (1.2), namely $p > \frac{N+2}{N-2}$, Sobolev embedding no longer holds, so that variational construction of solutions becomes difficult. However, del Pino-Musso-Pistoia investigated the nearly supercritical case in [14]. More precisely, they considered

$$\begin{cases} -\Delta u + u = u^{\frac{N+2}{N-2}+\epsilon} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.3}$$

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