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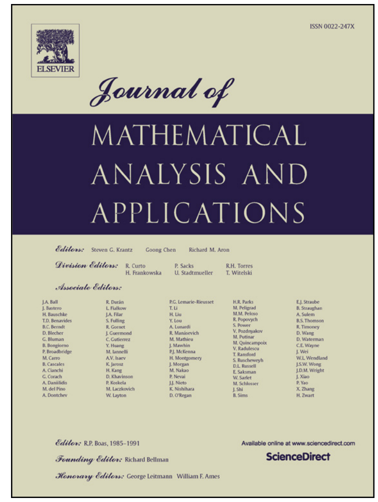


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# ON RANK-ONE ASYMMETRIC TRUNCATED TOEPLITZ OPERATORS ON FINITE-DIMENSIONAL MODEL SPACES

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Abstract. In this paper we present some consequences of the description of matrix representations of asymmetric truncated Toeplitz operators acting between finite-dimensional model spaces. We then describe all rank-one asymmetric truncated Toeplitz operators acting between finite-dimensional model spaces.

## 1. Introduction

Let $H^2$ be the classical Hardy space of the unit disk $\mathbb{D} = \{z\colon |z| < 1\}$ and let $P$ denote the Szegö projection, that is, the orthogonal projection from $L^2(\partial\mathbb{D})$ onto $H^2$.

The unilateral shift $S : H^2 \to H^2$ is defined on $H^2$ by

$$Sf(z) = zf(z).$$

It is known that the $S$-invariant subspaces of $H^2$ can be characterized in terms of inner functions. An inner function $\alpha$ is a function from $H^\infty$, the algebra of bounded analytic functions on $\mathbb{D}$, such that $|\alpha| = 1$ a.e. on $\partial\mathbb{D}$. The theorem of A. Beurling states that every closed nontrivial $S$-invariant subspace of $H^2$ is of the form $\alpha H^2$ for some inner function $\alpha$. Consequently, each of the closed nontrivial subspaces of $H^2$ invariant under the backward shift $S^*$ is given by

$$K_\alpha = H^2 \ominus \alpha H^2$$

with $\alpha$ inner. The space $K_\alpha$ is called the model space corresponding to the inner function $\alpha$.

The model space $K_\alpha$ has a reproducing kernel property and for each $w \in \mathbb{D}$ the function

$$k_w^\alpha(z) = \frac{1 - \overline{\alpha(w)}\alpha(z)}{1 - \overline{w}z}, \quad z \in \mathbb{D},$$

is the corresponding kernel function, that is, $f(w) = \langle f, k_w^\alpha\rangle$ for every $f \in K_\alpha$ ($\langle\cdot,\cdot\rangle$ being the usual integral inner product). Since $k_w^\alpha \in H^\infty$, the set $K_\alpha^\infty$ of all bounded functions in $K_\alpha$ is dense in $K_\alpha$.

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