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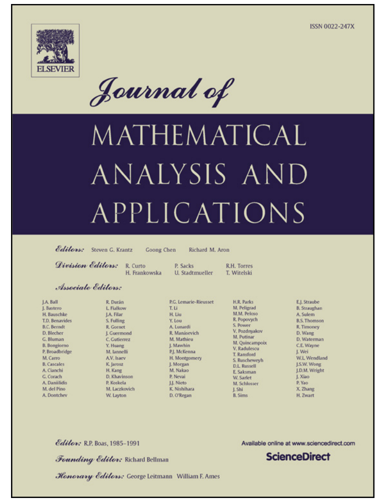

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Renormings of c_0 and the Fixed Point Property

Juan M. Álvaro, Pilar Cembranos and José Mendoza *
Departamento de Análisis Matemático
Universidad Complutense de Madrid
28040 Madrid (Spain)

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Abstract

It is unknown whether all renormings of c_0 fail the fixed point property (FPP) or not. In this note we give a sufficient condition for a renorming of c_0 to fail the FPP which is more general than the previously known ones. In fact the condition is satisfied for all renormings of c_0 we know. It also allows us to show that for some usual renormings of c_0 not only the whole space but all its infinite dimensional subspaces fail the FPP.

1 Introduction

Let c_0 be, as usual, the vector space of all null-sequences of real numbers. We denote by $\|\cdot\|_\infty$ the sup norm, defined by

$$\|(t_n)\|_\infty = \sup\{|t_n| : n \in \mathbb{N}\}$$

for all $(t_n) \in c_0$.

Let C be a nonempty subset of a Banach space $(X, \|\cdot\|)$, a mapping $T : C \to C$ is called *nonexpansive* if for any $x, y \in C$ we have

$$\|Tx - Ty\| \leq \|x - y\|.$$

The space $(X, \|\cdot\|)$ is said to have the *fixed point property* FPP if every nonexpansive mapping defined from a nonempty, closed, bounded and convex subset of X into itself has a fixed point. It is well known that $(c_0, \|.\|_\infty)$ fails the FPP and it was posed some time ago the following problem: *is there any renorming of c_0 enjoying the FPP?* No answer to this question is known yet, although the answer to the analogous question for ℓ_1 was obtained in [7]. To give an answer it seems interesting to find sufficient conditions, as general as possible, for a renorming to fail the FPP. In [2, 3, 4] some results in this direction can be found. In this note we give a very simple sufficient condition (see Theorem 2) which, as far as we know, is the most general that can be found in the literature. In fact all renormings of c_0 we know satisfy this condition (see the final section). In this way we can summarize in this note all known results on the subject, and we can also provide some new results.

All Banach spaces considered here are real and the notation is standard as in [8].

*e-mail: mid-nor@hotmail.com, Pilar_Cembranos@mat.ucm.es, Jose_Mendoza@mat.ucm.es