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Pricing Bermudan options under local Lévy models with default

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ABSTRACT

We consider a defaultable asset whose risk-neutral pricing dynamics are described by an exponential Lévy-type martingale. This class of models allows for a local volatility, local default intensity and a locally dependent Lévy measure. We present a pricing method for Bermudan options based on an analytical approximation of the characteristic function combined with the COS method. Due to a special form of the obtained characteristic function the price can be computed using a fast Fourier transform-based algorithm resulting in a fast and accurate calculation. The Greeks can be computed at almost no additional computational cost. Error bounds for the approximation of the characteristic function as well as for the total option price are given.

1. Introduction

In financial mathematics, the fast and accurate pricing of financial derivatives is an important branch of research. Depending on the type of financial derivative, the mathematical task is essentially the computation of integrals, and this sometimes needs to be performed in a recursive way in a time-wise direction. For many stochastic processes that model the financial assets, these integrals can be most efficiently computed in the Fourier domain. However, for some relevant and recent stochastic models the Fourier domain computations are not at all straightforward, as these computations rely on the availability of the characteristic function of the stochastic process (read: the Fourier transform of the transitional probability distribution), which is not known. This is especially true for state-dependent asset price processes, and for asset processes that include the notion of default in their definition. With the derivations and techniques in the present paper we make available the highly efficient pricing of so-called Bermudan options to the above mentioned classes of state-dependent asset dynamics, including jumps in asset prices and the possibility of default. In this

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sense, the class of asset models for which Fourier option pricing is highly efficient increases by the contents of the present paper. Essentially, we approximate the characteristic function by an advanced Taylor-based expansion in such a way that the resulting characteristic function exhibits favorable properties for the pricing methods.

Fourier methods have often been among the winners in option pricing competitions such as BENCHOP [16]. In [5], a Fourier method called the COS method, as introduced in [4], was extended to the pricing of Bermudan options. The computational efficiency of the method was based on a specific structure of the characteristic function allowing to use the fast Fourier transform (FFT) for calculating the continuation value of the option. Fourier methods can readily be applied to solving problems under asset price dynamics for which the characteristic function is available. This is the case for exponential Lévy models, such as the Merton model developed in [13], the Variance-Gamma model developed in [12], but also for the Heston model [6]. However, in the case of local volatility, default and state-dependent jump measures there is no closed form characteristic function available and the COS method cannot be readily applied.

Recently, in [14] the so-called *adjoint expansion method* for the approximation of the characteristic function in local Lévy models is presented. This method is worked out in the Fourier space by considering the adjoint formulation of the pricing problem, that is using a backward parametrix expansion as was also later done in [1]. In this paper we generalize this method to include a defaultable asset whose risk-neutral pricing dynamics are described by an exponential Lévy-type martingale with a state-dependent jump measure, as has also been considered in [11] and in [7].

Having obtained the analytical approximation for the characteristic function we combine this with the COS method for Bermudan options. We show that this analytical formula for the characteristic function still possesses a structure that allows the use of a FFT-based method in order to calculate the continuation value. This results in an efficient and accurate computation of the Bermudan option value and of the Greeks. The characteristic function approximation used in the COS method is already very accurate for the 2nd-order approximation, meaning that the explicit formulas are simple and this makes method easy and quick to implement. We prove error bounds for the 0th- and 1st-order approximation, justifying the accuracy of the method and present a wide range of numerical examples, showing the flexibility, accuracy and speed of the method.

2. General framework

We consider a defaultable asset S whose risk-neutral dynamics are given by:

$$\begin{aligned} S_t &= \mathbb{1}_{\{t<\zeta\}} e^{X_t}, \\ dX_t &= \mu(t, X_t)dt + \sigma(t, X_t)dW_t + \int_{\mathbb{R}} d\tilde{N}_t(t, X_{t-}, dz)z, \\ d\tilde{N}_t(t, X_{t-}, dz) &= dN_t(t, X_{t-}, dz) - \nu(t, X_{t-}, dz)dt, \\ \zeta &= \inf\{t \geq 0 : \int_0^t \gamma(s, X_s)ds \geq \varepsilon\}, \end{aligned} \tag{2.1}$$

where $\tilde{N}_t(t, x, dz)$ is a compensated random measure with state-dependent Lévy measure $\nu(t, x, dz)$. The default time ζ of S is defined in a canonical way as the first arrival time of a doubly stochastic Poisson process with local intensity function $\gamma(t, x) \geq 0$, and $\varepsilon \sim \text{Exp}(1)$ and is independent of X. Thus the model features: