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Boundedness and global asymptotic stability of constant equilibria in a fully parabolic chemotaxis system with nonlinear logistic source

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ABSTRACT

This paper deals with a fully parabolic chemotaxis system with nonlinear logistic source

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + u(1-\mu u^{r-1}), \\ v_t = \Delta v - v + u, \end{cases} \tag{0.1}$$

under homogeneous Neumann boundary conditions in a smooth bounded convex domain $\mathbb{R}^N$, with parameters $\mu, \chi > 0, r \geq 2$. It is shown that if $r > 2$ or $r = 2$ and $\mu > \frac{N\chi}{4}$, then for all sufficiently smooth initial data, the associated initial–boundary-value problem (0.1) possesses a unique global-in-time classical solution that is bounded in $\Omega \times (0,\infty)$, which satisfies

$$\limsup_{t\to\infty} \|u(\cdot,t)\|_{L^\infty(\Omega)} \leq \frac{\mu(\max_{t\geq 0}(\frac{N\chi}{4\mu}t^2 + \frac{r}{\mu}t - t^r))}{\min\{r-1,2\}}.$$

Moreover, with the assumption $u_0 \not\equiv 0$ and appropriate growth assumptions, the globally asymptotical stability of $((\frac{1}{\mu})^{\frac{1}{r-1}}, (\frac{1}{\mu})^{\frac{1}{r-1}})$ is established.

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1. Introduction

In this paper, we study the following Keller–Segel system with nonlinear logistic source

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla v) + \xi u(1 - \mu u^{r-1}), & x \in \Omega, t > 0, \\ v_t = \Delta v - v + u, & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0, & x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x), v(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where $r \geq 2, \xi, \mu \geq 0$, $\Omega \subset \mathbb{R}^N (N \geq 1)$ is a bounded convex domain with smooth boundary $\partial\Omega$, $\Delta = \sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2}$ and $\frac{\partial}{\partial \nu}$ denotes the outward normal derivative on $\partial\Omega$, $\chi > 0$ is a parameter referred to as chemosensitivity, $u = u(x,t)$ and $v = v(x,t)$ denote the density of the cells' population and the concentration of the chemoattractant, respectively.

If $\xi = 0$, (1.1) is an extended version of the well-known Keller–Segel system [12,13]. In the last 40 years, after the pioneering works of Keller and Segel [12,13], chemotaxis has been described by using nonlinear systems of PDEs with second order terms modeling the aggregation of the organisms. From a theoretical point of view, the problem presents important mathematical challenges. Some of these challenges are described by the mechanism which drives the system to finite time blow-up (see Cieślak et al. [2–4], Horstmann et al. [8,9]) or to global boundedness (Keller and Segel [11,12]) and to obtain the constraints of the parameters, the threshold values which decide the behavior and the stability of the system [5,16,25]. In that direction many authors have studied the qualitative properties of these mathematical models depending on the relations between such parameters and the initial data. It is well known that the model has only bounded solutions if $N = 1$ ([17]); if $N = 2$, there exists a threshold value for the initial mass that decides whether the solutions can blow up or exist globally in time [7,8,24]; when $N \geq 3$, there is no such threshold [3,24,25,27]. Moreover, for more general diffusion function and chemotactic sensitivity, many results relate the question of whether the solutions are bounded or blow-up (see e.g. Winkler et al. [1,27,18], Ishida et al. [10], Cieślak and Stinner [3]).

The influence of logistic sources for the classical Keller–Segel system is significant, since the mass conservation is not true anymore with $\xi \neq 0$. Generally, the logistic-type growth restrictions would benefit the global existence and boundedness of solutions to the Keller–Segel models. In particular, for the classical logistic source ($r = 2$), the problem has been studied extensively by many authors (see e.g., Tello and Winkler [20], Winkler et al. [1,23,29]). In [20], Tello and Winkler discussed the existence of global bounded classical solutions to the parabolic–elliptic model under the assumption that either $N \leq 2$, or that the logistic damping effect $\mu > \frac{N-2}{N}\chi$. In [26], Winkler proved the parabolic–parabolic model (1.1) is global and bounded provided that μ is sufficiently large. Furthermore, for a more general logistic source ($r > 1$), there have been many papers which dealt with the question of whether the solutions are global bounded or blow-up (see Galakhov et al. [6], Wang et al. [22,21], Winkler [29], Zheng [30,31]).

Going beyond these boundedness statements, a number of results are available which show that the interplay of chemotactic cross-diffusion and cell kinetics of (classical) logistic-type may lead to quite colorful dynamics (see e.g. Winkler et al. [19,29,29,28], Galakhov et al. [6]). For instance, for the classical logistic source ($r = 2$), Winkler [29] found that the solutions of one dimensional parabolic–elliptic models may become large at intermediate time scales provided that $\mu \geq 1$.

In [19], Tao and Winkler proved that the population of cells always persists when $\chi, \xi, \mu > 0$, i.e., for any nonnegative global classical solution (u, v) of (1.1) (if it exists) with $u \not\equiv 0$, there is $m > 0$ such that

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