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Dichotomies and asymptotic equivalence in alternately advanced and delayed differential systems

Aníbal Coronel [a,*], Christopher Maulén [b], Manuel Pinto [b], Daniel Sepúlveda [c]

[a] GMA, Departamento de Ciencias Básicas, Facultad de Ciencias, Universidad del Bío-Bío, Campus Fernando May, Chillán, Chile
[b] Departamento de Matemáticas, Facultad de Ciencias, Universidad de Chile, Chile
[c] Escuela de Matemática y Estadística, Facultad de Ciencias de la Educación, Universidad Central de Chile, Chile

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ABSTRACT

In this paper, ordinary and exponential dichotomies are defined in differential equations with equations with piecewise constant argument of general type. We prove the asymptotic equivalence between the bounded solutions of a linear system and a perturbed system with integrable and bounded perturbations.

1. Introduction

The study of differential equations with piecewise constant argument is motivated by several applications coming from different fields of science and by their own mathematical definition as hybrid dynamical systems [2,9,29]. For a longer discussion on applications consult the references [5,10,21,22,27,36,37]. Here the meaning of hybrid is given in the sense that they combine the behavior of differential and difference equations. In general, the typical form of this kind of equations is given by the following functional equation

$$\mathbf{x}'(t) = \mathbf{F}(t, \mathbf{x}(t), \mathbf{x}(\gamma(t))), \tag{1}$$

where $\mathbf{x} : \mathbb{R} \to \mathbb{C}^p$ is the unknown function, $t \in \mathbb{R}$ usually denotes the time, $\mathbf{F}$ is a given function from $\mathbb{R} \times \mathbb{C}^p \times \mathbb{C}^p$ to $\mathbb{C}^p$, and γ is a given general step function in the sense that

* Corresponding author.
E-mail addresses: acoronel@ubiobio.cl (A. Coronel), christoph.maulen.math@gmail.cl (C. Maulén), pintoj.uchile@gmail.cl (M. Pinto), daniel.sep.oe@gmail.com (D. Sepúlveda).

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$$\left.\begin{array}{l}\gamma:\mathbb{R}\to\mathbb{R} \text{ is defined by } \gamma(t)=\zeta_i \text{ for } t\in I_i=[t_i,t_{i+1}), \text{ where } \{t_i\}_{i\in\mathbb{Z}}\\ \text{and } \{\zeta_i\}_{i\in\mathbb{Z}} \text{ are two given (fix) sequences such that } t_i\le\zeta_i\le t_{i+1}\\ \text{with } t_i<t_{i+1} \text{ for each } i\in\mathbb{Z} \text{ and } t_i\to\pm\infty \text{ when } i\to\pm\infty.\end{array}\right\} \tag{2}$$

The genesis of the study of this kind of functional equations goes back to the work of Myshkis [28], who proposed an equation of type (1)–(2) with the particular step functions $\gamma(t) = [t]$ and $\gamma(t) = 2\,[(t + 1)/2]$. Here $[\cdot]$ denotes the greatest integer function. By simplicity of the presentation, we use hereinafter the terms DEPCA and DEPCAG to refer the differential equations with piecewise constant argument when the step function is based on the greatest integer function and when the step function is of the general type given by (2), respectively. In particular, note that the equations studied by Myshkis are DEPCAs. Later, in the early 80's, a systemic analysis of (1)–(2) was introduced by Wiener and collaborators, see [1,17,39,40] and references therein. Afterwards, the contribution to the development of the theory was given by many authors see for instance [3–5,7,10,18,21–23,25,29,33,36,39–44]. Nowadays, there exist an intense and increasing interest to understand the qualitative behavior, to get novel applications and to solve numerically the equations, since a general theory for (1)–(2) is far to be closed, see [5,6,11–13,15,14,16,35,38].

An important point to observe is the notion of the solution or more generally the types of approach to analyze (1)–(2). Actually, generally speaking, the notion of solution for functional differential equations is one of the most important tasks. Now, we recall that the original notion of the integration (or solution) of a DEPCA was introduced in [1,17,40] and is based on the reduction to discrete equations. This approach presents some disadvantages when we require a generalization to analyze DEPCAGs. In particular, for instance to solve the Cauchy problem requires that the initial moments must be integers, see [40] for details. Another approach to study a general quasilinear DEPCAG was introduced by Akhmet [3,4], and is based on the construction of an equivalent integral equation and remarking the clear influence of the discrete part. The methodology of Akhmet permits to overpass the difficulties of the methodology of Wiener and collaborators. Moreover, Akhmet adapts the notion of solution given by Wiener and used previously by Papaschinopoulos to study a particular type of DEPCA, see [29]. The notion of Akhmet solution is given in terms of continuity of the solution on each t_i, the existence of the derivatives on each t with possible exception of some t_i and the local satisfaction of the equation, see Definition 2.1 below. Then, in spite of its functional character a quasilinear DEPCAG has similar properties to ordinary differential equations. For further details, consult for instance [8,20,21,23,24,35]. Here, in this paper, we use the approach of Akhmet. Thus, we do not need to impose any restrictions on the discrete equations and we assume more easily verifiable conditions on the coefficients, similar to those for ordinary differential equations.

In this paper, we are interested in the asymptotic equivalence of some DEPCAGs. Now, in order to precise the different type of systems which will be used in the paper, we introduce a particular notation of each case. Indeed, throughout the paper, we consider that x, z, u, y, w, v satisfy the following particular cases of (1)–(2):

$$x'(t) = A(t)x(t), \tag{3}$$

$$z'(t) = A(t)z(t) + B(t)z(\gamma(t)), \tag{4}$$

$$u'(t) = B(t)u(\gamma(t)), \tag{5}$$

$$y'(t) = A(t)y(t) + B(t)y(\gamma(t)) + g(t), \tag{6}$$

$$w'(t) = A(t)w(t) + B(t)w(\gamma(t)) + f(t, w(t), w(\gamma(t))), \tag{7}$$

$$v'(t) = A(t)v(t) + B(t)v(\gamma(t)) + g(t) + f(t, v(t), v(\gamma(t))). \tag{8}$$