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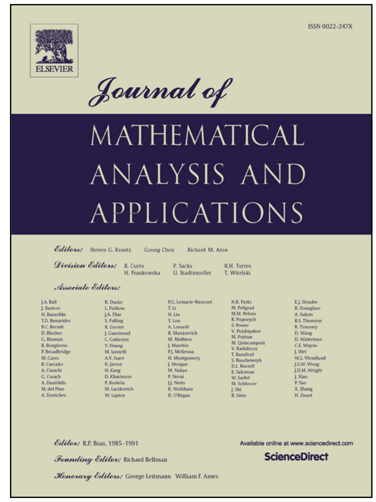

Nonuniform Sampling and Recovery of Bandlimited Functions in Higher Dimensions

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NONUNIFORM SAMPLING AND RECOVERY OF BANDLIMITED FUNCTIONS IN HIGHER DIMENSIONS

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ABSTRACT. We provide sufficient conditions on a family of functions $(\phi_\alpha)_{\alpha\in A}:\mathbb{R}^d\to\mathbb{R}$ for sampling of multivariate bandlimited functions at certain nonuniform sequences of points in $\mathbb{R}^d$. We consider interpolation of functions whose Fourier transform is supported in some small ball in $\mathbb{R}^d$ at scattered points $(x_j)_{j\in\mathbb{N}}$ such that the complex exponentials $\left(e^{-i\langle x_j,\cdot\rangle}\right)_{j\in\mathbb{N}}$ form a Riesz basis for the L_2 space of a convex body containing the ball. Recovery results as well as corresponding approximation orders in terms of the parameter α are obtained.

1. Introduction

The theory of interpolation has long been of interest to approximation theorists, and has connections with many areas of mathematics including harmonic analysis, signal processing, and sampling theory (to name just a few). The theory of spline interpolation at the integer lattice was championed by I.J. Schoenberg, and typically falls under the heading of "cardinal spline interpolation." More generally, cardinal interpolation schemes are those in which a given target function is interpolated at the multi-integer lattice in $\mathbb{R}^d$. Some study has been made of the connection between cardinal interpolation, sampling theory of bandlimited functions, and radial basis function theory. Schoenberg himself showed that bandlimited functions can be recovered by their cardinal spline interpolants in a limiting sense as the order of the spline tends to infinity, and similar analysis shows that such functions can also be recovered by cardinal Gaussian and multiquadric interpolants.

Lately, the ideas of Schoenberg and many of his successors have been used to tackle problems in a broader setting, namely interpolation schemes at infinite point-sets that are nonuniform. The richness of the theory for lattices suggests a search for comparable results in the nonuniform setting. To that end, Lyubarskii and Madych [16] considered univariate bandlimited function interpolation and recovery by splines, thus extending Schoenberg's ideas to the nonuniform setting. Inspired by their work, Schlumprecht and Sivakumar [19] showed analogous recovery results using translates of the Gaussian kernel. Then Ledford [14] gave sufficient conditions on a family of functions to yield similar convergence results for bandlimited functions in one dimension. One of the unifying themes in these works is the use of a special structure on the points, namely that they form *Riesz-basis sequences* (or complete interpolating sequences) for an associated Paley–Wiener space, or equivalently, the corresponding sequence of complex exponential functions forms a Riesz basis for a certain L_2 space. We will discuss this in more detail later, but the main point here is that in one dimension, such sequences are characterized and relatively easy to come by. However, in higher dimensions, the problem becomes significantly more complicated as the existence of such Riesz-basis sequences is unknown even for nice domains such as the Euclidean ball.

2000 *Mathematics Subject Classification.* Primary 41A05, 41A30, Secondary 42C30.

Key words and phrases. Nonuniform Sampling; Multivariate bandlimited functions; Radial basis function interpolation; Approximation rates.

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