J. Math. Anal. Appl. 453 (2017) 954–972

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Linear electron stability for a bi-kinetic sheath model

Mehdi Badsi

UPMC-Paris06, CNRS UMR 7598, Laboratoire Jacques Louis Lions, 4 pl. Jussieu, F75252 Paris Cedex 05, France

ARTICLE INFO

Article history:
Received 19 May 2016
Available online 28 April 2017
Submitted by H. Liu

Keywords:
Plasma wall interaction
Debye sheath
Kinetic equations
Vlasov–Poisson–Ampère system
Linear stability
Degenerate transport equations

ABSTRACT

We establish the linear stability of an electron equilibrium for an electrostatic and collisionless plasma in interaction with a wall. The equilibrium we focus on is called in plasma physics a Debye sheath. Specifically, we consider a two species (ions and electrons) Vlasov–Poisson–Ampère system in a bounded and one dimensional geometry. The interaction between the plasma and the wall is modeled by original boundary conditions: On the one hand, ions are absorbed by the wall while electrons are partially re-emitted. On the other hand, the electric field at the wall is induced by the accumulation of charged particles at the wall. These boundary conditions ensure the compatibility with the Maxwell–Ampère equation. A global existence, uniqueness and stability result for the linearized system is proven. The main difficulty lies in the fact that (due to the absorbing boundary conditions) the equilibrium is a discontinuous function of the particle energy, which results in a linearized system that contains a degenerate transport equation at the border.

1. Introduction

1.1. A kinetic model of plasma-wall dynamics: the Vlasov–Poisson–Ampère system

We consider an electrostatic and collisionless plasma consisting of one species of ions and electrons. We use a kinetic approach to model this plasma. To this purpose, we set $\Omega = (0,1) \times \mathbb{R}$ and denote by $(x,v) \in \overline{\Omega} = [0,1] \times \mathbb{R}$ the phase space variable, where x is the particle position and v the particle velocity. This work is concerned with the linear stability of an equilibrium for the two species Vlasov–Poisson system in the presence of spatial boundaries

$$\begin{cases} \partial_t f_i + v\partial_x f_i + E\partial_v f_i = 0 & \text{in} \quad (0,+\infty) \times \Omega, \\ \partial_t f_e + v\partial_x f_e - \frac{1}{\mu} E\partial_v f_e = 0 & \text{in} \quad (0,+\infty) \times \Omega, \end{cases} \tag{1}$$

$$-\varepsilon^2 \partial_{xx}\phi = n_i - n_e \quad \text{in} \quad (0,+\infty) \times (0,1), \tag{2}$$

E-mail address: badsi@ljll.math.upmc.fr.

http://dx.doi.org/10.1016/j.jmaa.2017.04.055

where $f_i : [0,+\infty) \times \overline{\Omega} \to \mathbb{R}^+$, $f_e : [0,+\infty) \times \overline{\Omega} \to \mathbb{R}^+$ are the ions and electrons distribution functions in the phase-space and $\phi : [0,+\infty) \times [0,1] \to \mathbb{R}$ is the electric potential. Here the physical parameters μ and ε stand respectively for the mass ratio between electrons and ions, and a normalized Debye length that will be (for simplicity) in the sequel taken equal to 1. We also denote

$$E = -\partial_x \phi, \quad n_i = \int_{\mathbb{R}} f_i dv, \quad n_e = \int_{\mathbb{R}} f_e dv,$$

the electric field, the ion density and the electron density. The boundary conditions are given for all $t \in (0,+\infty)$ by

$$\begin{cases} f_i(t,0,v>0) = f_i^{in}(v), & f_i(t,1,v<0) = 0, \\ f_e(t,0,v>0) = f_e^{in}(v), & f_e(t,1,v<0) = \alpha f_e(t,1,-v), \end{cases} \tag{3}$$

$$\phi(t,0) = 0, \quad E(t,1) = E^*(t,f_i,f_e), \tag{4}$$

where f_i^{in} and f_e^{in} denote two given incoming particles velocity distributions that are time independent. The scalar parameter α belongs to the interval $[0,1)$ and represents the rate of re-emitted electrons in the domain $(0,1)$. The scalar $E^*(t,f_i,f_e)$ depends on the unknown (f_i,f_e,ϕ) via the formula

$$E^*(t,f_i,f_e) = \left(E_w^0 - \int_0^t \int_{\mathbb{R}} \left(f_i(\tau,1,v) - f_e(\tau,1,v)\right) v dv d\tau \right). \tag{5}$$

Up to our knowledge, the theory of existence and uniqueness for such a initial boundary value problem (1)–(4) has not been treated in full details. In the one dimensional case, there is the result of Bostan [5] which establishes the existence and uniqueness of the mild solution to a Vlasov–Poisson system in which the boundary conditions do not depend on the solution itself. Still in the one dimensional case, the work of Guo [10] studies the dynamic of a plane diode. Also, the result of BenAbdallah [3] shows the existence of weak solutions for the Vlasov–Poisson system in dimension greater than or equal to one, but once again the boundaries are not coupled to the solution itself. The existence and uniqueness of weak solutions in the half-space with specular reflection condition is obtained in [11]. The case of partially absorbing boundary condition is treated in [9]. The existence of a stationary solution to the system (1)–(4) was proven in [1], the stationary solution corresponds to the Debye sheath (see [17] for further physical details). This work can be considered as a continuation of the work [1], and a first step in the study of the wellposedness of the non-linear system (1)–(4).

1.2. Physical interpretation of the model

The bi-kinetic model (1)–(4) models the dynamical transition between the core of a plasma and a wall (see for instance [13]). The region of plasma we consider is modeled by the line segment $[0,1]$ where $x = 0$ is assumed to be somewhere in the bulk plasma and thus a source of particles. The sources here are modeled by the injection of particles that are mathematically encoded in the given distributions f_i^{in} and f_e^{in}. The wall at $x = 1$ is supposed to be metallic and partially absorbing: it absorbs completely the ions and re-emits a fraction α of the electrons. The parameter α can be seen as a constitutive parameter of the wall. The accumulation of charged particles at the wall induces an electric-field that is given by (5) (the number E_w^0 denotes the initial electric field at the wall). The boundary condition of the electric-field at the wall can be formally re-written as $\partial_t E(t,1) + j(t,1) = 0$ where $j(t,1) := \int_{\mathbb{R}} (f_i(t,1,v) - f_e(t,1,v)) v dv$ is the current density at the wall. At a formal level, one easily verifies that this boundary condition ensures the