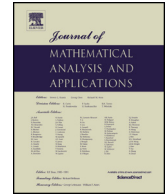




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Extensions of subcopulas

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ABSTRACT

In view of Sklar's Theorem the probability distribution function of every (not necessarily continuous) random vector can be uniquely decomposed in terms of the marginal distributions of its components and a suitable subcopula. The study of such latter functions is therefore of interest for understanding the dependence information of non-continuous variables. Here, we investigate some analytical properties of the class of subcopulas, including compactness (with respect to a novel metric), approximations and Baire category results. Moreover, under a suitable assumption, we describe all possible extensions from a subcopula to a copula in any dimension.

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1. Introduction

Copulas have been largely employed for describing the association of continuous random variables, as can be witnessed by several contributions devoted to the topic; see, for instance, [11,18,21,25]. Another related aspect of interest is, nowadays, the use of copulas in the determination of statistical models for a random vector $\mathbf{X} = (X_1, \dots, X_p)$ whose components are possibly not-continuous. In this latter case, however, the copula associated with \mathbf{X} is not anymore unique, a fact that needs special care in several practical problems (see, for instance, [16,22]). In fact, as can be inferred by the proof of Sklar's Theorem [31] (see also [9,12,30]), a copula associated with a non-continuous random vector \mathbf{X} is uniquely determined only on the Cartesian product of the (closure of the) ranges of X_1, \dots, X_p , but various extensions to the whole domain $[0, 1]^p$ are possible. In [31], the term *subcopula* was introduced to denote the function (defined on a suitable subset of $[0, 1]^p$) that contains the information about the dependence of a not-necessarily continuous random vector.

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Various methods to extend a subcopula to a copula have been considered, for instance, in [6,31], while maximal and minimal extensions can be found in [4,16]. Such methods are relevant, for instance, in defining various measures of association for non-continuous data [24,26,33]. Moreover, they can provide tools for nonparametric estimation of a copula, where it is of interest to smooth the empirical copula while preserving copula properties [13,17,27,32], and/or extend copulas defined on a discrete setting [20,23,29]. Moreover, extensions of sub-copulas may be helpful to understand the limit of copula-based inferential procedures when they are applied, without some due changes, to non-continuous data [3,19,28].

Here, we continue the study of subcopulas and their extensions by providing a general framework to deal with such problems. Specifically, we introduce a distance ξ in the class of subcopulas that is based on the Hausdorff distance of the respective graphs. As a relevant aspect, we show that the class of subcopulas equipped with the topology induced by ξ is compact (Section 2). Hence, we use continuity arguments to prove, in an alternative way, that any subcopula can be extended to a copula (Section 2.1). Finally, in Section 3 we provide the general analytical expression for all the extensions of a subcopula in a multivariate setting, which generalizes the results presented in [6] for the bivariate case.

2. A metric for subcopulas

For basic definitions and properties of copulas we refer to [11,25]. Here, we only recall the minimum bare that is necessary to make this manuscript self-contained.

Definition 2.1. Let A_1, \dots, A_p be subsets of $[0, 1]$ containing both 0 and 1. Then a *subcopula* is a function $S : A_1 \times \dots \times A_p \rightarrow [0, 1]$ such that

- (a) $S(u_1, \dots, u_p) = 0$ if $u_j = 0$ for at least one index $j \in \{1, \dots, p\}$;
- (b) $S(1, \dots, 1, t, 1, \dots, 1) = t$ for every $t \in A_j$ ($j \in \{1, \dots, p\}$);
- (c) For every rectangle $[\mathbf{a}, \mathbf{b}]$ having its vertices in $A_1 \times \dots \times A_p$, the S -volume of $[\mathbf{a}, \mathbf{b}]$ is non-negative, namely $V_S([\mathbf{a}, \mathbf{b}]) \geq 0$, where

$$V_S([\mathbf{a}, \mathbf{b}]) = \sum_{\mathbf{v} \in \text{ver}[\mathbf{a}, \mathbf{b}]} \text{sign}(\mathbf{v}) S(\mathbf{v}),$$

with

$$\text{sign}(\mathbf{v}) = \begin{cases} 1, & \text{if } v_j = a_j \text{ for an even number of indices,} \\ -1, & \text{if } v_j = a_j \text{ for an odd number of indices,} \end{cases}$$

and $\text{ver}([\mathbf{a}, \mathbf{b}]) = \{a_1, b_1\} \times \dots \times \{a_p, b_p\}$ is the set of vertices of $[\mathbf{a}, \mathbf{b}]$.

The class of subcopulas will be denoted by \mathcal{S} . Every $S \in \mathcal{S}$ is Lipschitz continuous with constant 1 (shortly 1-Lipschitz), i.e.

$$|S(\mathbf{u}) - S(\mathbf{v})| \leq \sum_{i=1}^p |u_i - v_i| \tag{1}$$

for every $\mathbf{u}, \mathbf{v} \in \text{Dom}(S)$. Thus, the domain of S can be extended without loss of generality to its closure. In the following, if not otherwise stated, we will therefore assume that A_1, \dots, A_p are closed.

A *copula* is a subcopula defined on $[0, 1]^p$, namely such that $A_j = [0, 1]$ for every $j \in \{1, \dots, p\}$. The class of copulas will be denoted by \mathcal{C} . In particular, given $S \in \mathcal{S}$, a copula C is said to be an *extension*

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